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THE FILTERING OF LUMINANCE AND CHROMINANCE SIGNALS TO AVOID CROSS-COLOUR IN A PAL COLOUR SYSTEM

J.O. Drewery, M.A., Ph.D.

Summary

The origins of cross-colour within the PAL system are examined with the aid of one, two and three-dimensional models. Analysis shows where the luminance and chrominance energy is concentrated assuming normal scene content. Filters are then developed which restrict the luminance and chrominance energies to distinctly separate regions so minimising cross-colour. These filters range from the simplest to the most complex forms and the degradations so introduced are discussed. Elimination of cross-colour is only obtained at the expense of losing spatial or temporal resolution. The conclusion is reached that with the most complex forms of processing a resolution considerably better than that at present can be achieved with no cross-colour.

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1. Introduction

The PAL colour system, in common with the NTSC and SECAM systems compresses luminance and chrominance information into the spectral space occupied, in a monochrome system, by the luminance alone. It is therefore inevitable that unless the luminance and chrominance are spectrally shaped before being combined it will be very difficult to separate them without incurring mutual interference. There is thus a straight choice between cross-colour and loss of spatial or temporal resolution. It is the purpose of this study to examine this choice in more detail and to suggest a possible improvement on the present system.

2. Basic conditions for separability

The present method of transmitting luminance and chrominance using the PAL system is shown in Fig. 1. The corresponding spectra are shown in Fig. 2. The PAL specification of chrominance bandwidth is fairly lax in that the −3 dB point must be higher than 1·3 MHz but the −20 dB point can be as high as 4 MHz. In practice the chrominance is low-pass filtered before modulating the subcarrier at the coder to produce the typical spectrum of Fig. 2. As the luminance is usually unfiltered, chrominance and luminance therefore share the same spectral space in the region around subcarrier frequency.

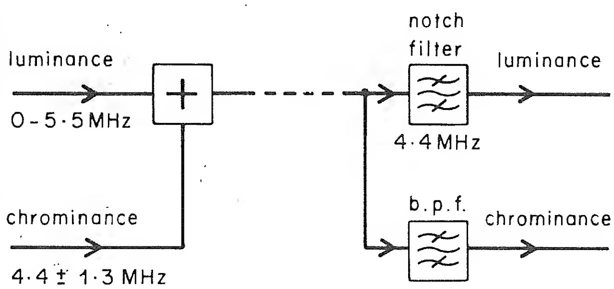


Fig. 1 - Present method of transmitting luminance and chrominance using the PAL system

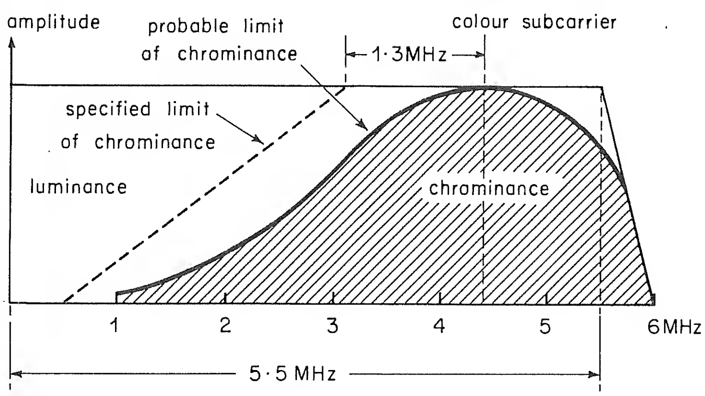


Fig. 2 - The spectra of luminance and chrominance transmitted by the PAL system

At the decoder, the chrominance path contains a bandpass filter centred on subcarrier frequency with a fairly gentle cut having a typical bandwidth of ±1·6 MHz at −6 dB. Thus the chrominance path also accepts all the luminance in this region, which gives rise to cross-colour. Equally the luminance path contains a notch filter centred on subcarrier frequency of typical width ±500 kHz. Thus although the subcarrier frequency and some of its sidebands are rejected, the remaining chrominance sidebands are accepted by the luminance circuit and cause cross-luminance. This appears as moving dots on coloured edges.

Assuming that both luminance and chrominance must, between them, occupy the 5·5 MHz band, a way of eliminating these cross-signals is to confine the luminance and chrominance to distinctly separate bands within the total range. They can then be separated at the decoder. Such a matched coding and decoding system is shown in Fig. 3. F is any filter with an ideal spectral characteristic assuming values of zero or infinite attenuation. In the coder the addition of negative luminance to chrominance in all regions where chrominance is passed by the filter gives complementary luminance and chrominance spectra and ensures perfect separation. At the decoder the matched filter passes only the chrominance so that the residue must be pure luminance.

An advantage of the matched, complementary system of Fig. 3 is that only one filter characterises the system. A disadvantage is that with practical filters there will be a spectral region or regions where the voltage response is a half, giving rise to cross-over points where the luminance and chrominance are attenuated equally by 6 dB. After separation at the decoder this means that at these points the cross-signal levels will rise to a minimum attenuation of

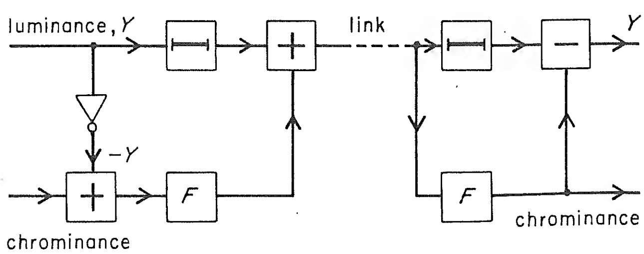


Fig. 3 - Matched complementary coding and decoding to eliminate cross-colour in PAL coded signals

12 dB.* The more perfect the filter, the smaller these regions become.

The structure of the spectral characteristic of F may be as simple or as complex as required. For example, F may be a simple bandpass filter extending from, say, 3·3 MHz to 5·5 MHz. In this case no luminance whatever above 3·3 MHz would be transmitted. Although this would solve the problem of cross-colour, it would be a solution negating the band-sharing philosophy of colour transmission systems. It is the purpose of this study to investigate more complex forms of F.

It is also of importance to discover the effect of using a filter at only one or other end of the system. For example, if the filter were costly it might be uneconomic to install it at the decoder. Again, economic considerations for internal signal distribution would be different from those for the link from transmitter to domestic receiver.

To discover forms of F that are likely to be more useful than a simple bandpass filter it is necessary to investigate the spectral structure of the existing, i.e. unfiltered PAL signal. However, as the form of F becomes more complex it is increasingly difficult to understand its effect by considering the purely one-dimensional spectrum of the signal. It is for this reason that the concepts of two- and three-dimensional spectra will be introduced. These will give a much clearer picture of what the more complex forms of F do. The following sections are therefore grouped under the headings of the one-, two- and three-dimensional approaches.

* If the spectral characteristic of the filter is $F(\omega)$, then the characteristic of the overall system for the chrominance is $F^2(\omega)$ and for the luminance $(1-F(\omega))^2$. At the decoder the characteristic of the luminance which enters the chrominance path is $1-F(\omega)$ and that of the chrominance which enters the luminance path is $F(\omega)$. So the cross-colour characteristic is $(1-F(\omega))\,F(\omega)$ and the cross-luminance characteristic is $F(\omega)\,(1-F(\omega))$. Thus the cross-signal characteristics are identical and have a maximum value of ¼ or −12 dB.

3. The one-dimensional approach

3.1. The spectrum of unprocessed PAL

The theory of Mertz and Gray[2] predicts that a spatial frequency having m cycles/picture width and n cycles/picture height,* (m, n) transforms into a signal frequency of ν cycles/second where ν is given by

$$\nu = mf_H + nf_V \qquad (1)$$

For a horizontal scanning system f_H is the line frequency, f_L, and for a sequential system f_V is the picture frequency. Then $f_L = Nf_V$ where the picture is scanned in N lines. For an interlaced system f_V is the field frequency and $f_H = Nf_V/2$ where, as before, a picture contains N lines. Thus Equation (1) becomes

$$\nu = f_L\,(m - n/N) \qquad (2a)$$

for a sequential system and

$$\nu = f_L\,(m - 2n/N) \qquad (2b)$$

for an interlaced system. (The negative sign occurs because the scanning proceeds in the negative y direction.)

The amplitude of the component at frequency ν is proportional to the amplitude of the spatial frequency (m, n) which transforms to it. Now statistically the average picture content is such that most of the energy is concentrated in components having long wavelengths, i.e. low values of m and n. Thus Equation (2a) shows that the signal spectral energy for a sequential system is concentrated in bunches centred on line harmonics, the bunches increasing in amplitude towards zero frequency. This is shown in Fig. 4.

The spatial frequencies $(m, N/2)$ correspond to the signal frequencies $(m - ½)f_L$ which lie midway between

* The picture width and height ignore blanking. This can be considered as a black border forming part of the picture.

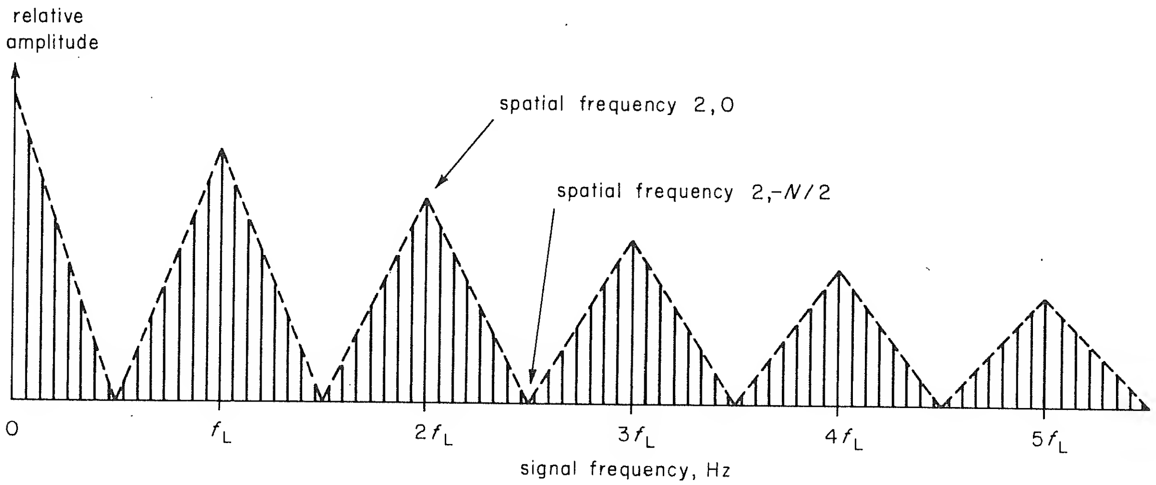


Fig. 4 - Luminance spectrum of typical sequentially scanned television signal

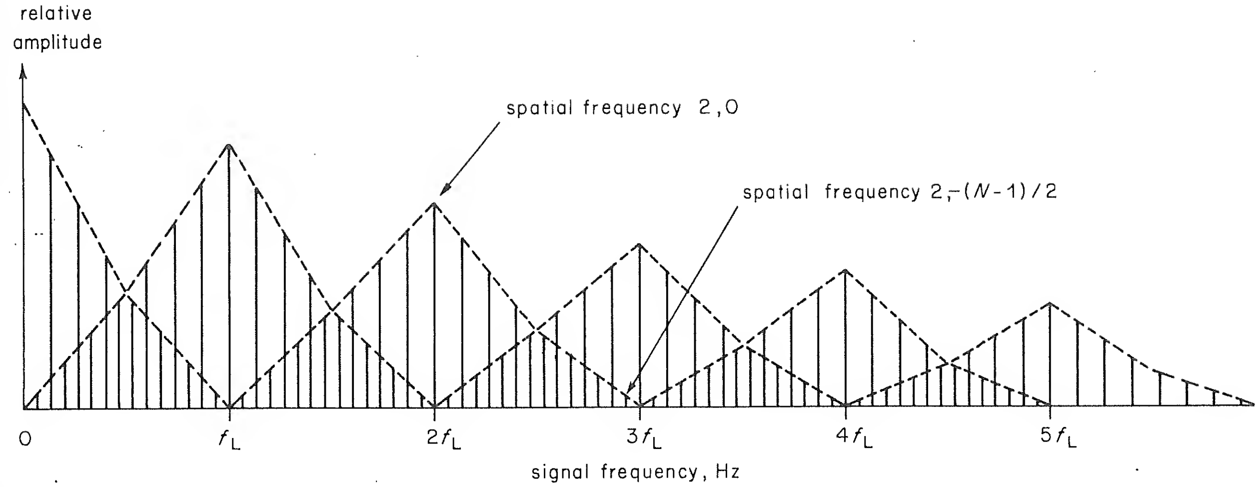


Fig. 5 - Luminance spectrum of typical interlaced scanned television signal

line harmonics. Spatial frequencies with values of n greater than $N/2$ thus transform to frequencies in adjoining bunches and are confused with spatial frequencies having a value of m differing by one. Now the scanning action, which transforms spatial to signal frequencies, samples the scene vertically at a rate of N samples per picture height. It is well known from sampling theory that a function must not contain components with frequencies higher than half the sampling frequency if aliasing is to be avoided. Thus the conditions for avoiding vertical aliasing and confusion are identical and are different ways of saying the same thing. Vertical frequencies can be restricted before scanning to values of n below $N/2$ by optical filtering or suitably shaping the scanning aperture.

For an interlaced system the same general conclusions apply. However, Equation (2b) shows that the signal frequencies corresponding to successive values of n, for the same value of m, are now separated by twice the value of that for a sequential system. Thus for the same range of values of n the bunches of energy now overlap as shown in Fig. 5. This overlapping does not, however, cause confusion because the bunches interleave. This is a necessary consequence of the fact that for an interlaced system N must be odd. In fact confusion cannot occur until n exceeds $N/2$ as before.

The spatial frequencies $(m, N/2-½)$ correspond to the signal frequencies $(m - 1 + 1/N)f_L$ which are close to line frequency harmonics. Thus the region near line harmonics corresponds to both zero and maximum vertical frequency components and the region midway between line harmonics corresponds to the vertical midband of the picture spatial spectrum where alternate field lines are, say, black and white.

The spectrum of the chrominance can be derived by separating U and V components. The U component is straightforward. The baseband U signal modulates the subcarrier using double sideband suppressed carrier modulation. The resulting baseband spectrum thus consists of bunches of energy centred on multiples of line frequency, as for luminance, but the whole structure is centred on and symmetrical about subcarrier frequency. As subcarrier frequency is less than 284 times line frequency by about one quarter of line frequency the bunches of U chrominance are likewise centred on frequencies with the same offset below multiples of line frequency as shown in Fig. 6.

The spectrum of the V chrominance is more complex because of the phase alternation. The baseband V signal is first multiplied by a switching function, which is a square wave of half line-frequency, before double-sideband suppressed-carrier modulation. The switching can be considered as double-sideband suppressed-carrier modulation of the square wave by the baseband V signal. As the spectrum

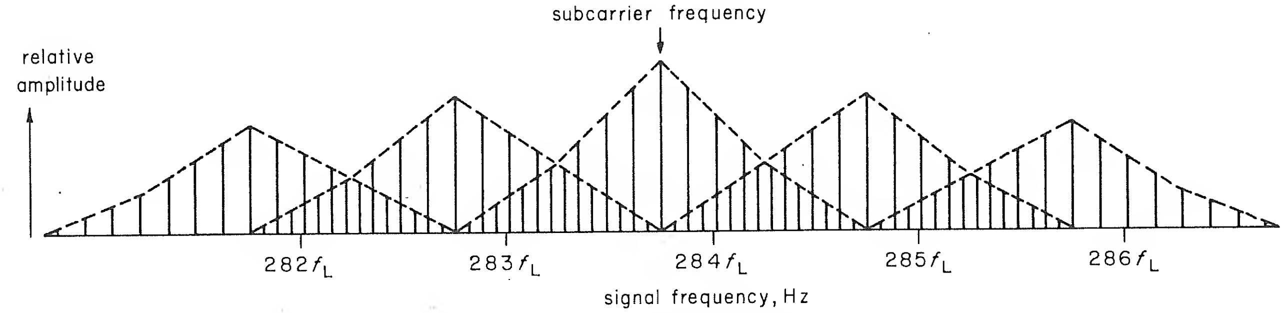


Fig. 6 - Spectrum of U chrominance signal

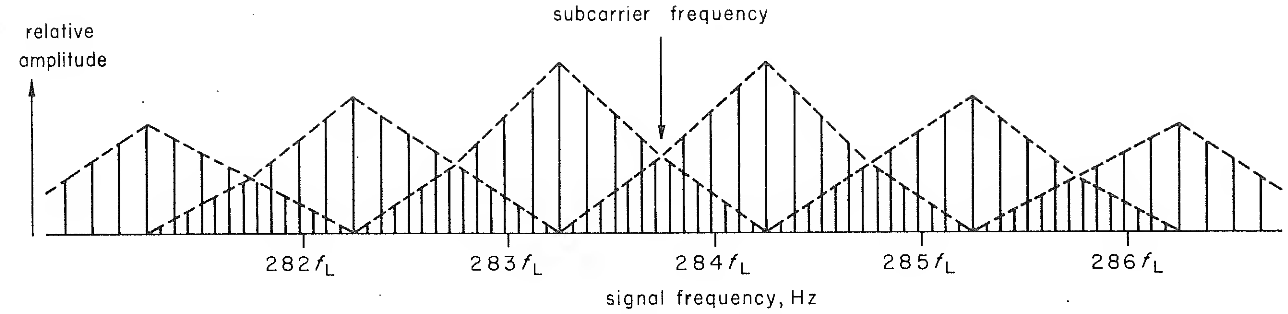


Fig. 7 - Spectrum of V chrominance signal

of the square wave consists of lines at odd multiples of half line-frequency having a $(\sin x)/x$ envelope, each of these lines becomes a carrier for the baseband V signal. The resulting spectrum is thus bunches of energy centred on odd multiples of half line-frequency. However, because the carrier frequencies are separated by exactly line frequency, bunches belonging to one carrier are exactly confused with bunches belonging to another. Thus the bunch centred on half line-frequency contains contributions from all baseband bunches with differing values of m. It is therefore not possible to associate a particular signal frequency with a unique pair (m, n) but only with a particular value of n. This confusion is, however, resolved on demodulation. Finally the switched baseband V signal is double-sideband suppressed-carrier modulated on the quadrature subcarrier so translating the whole structure spectrally to be centred on subcarrier frequency. The spectrum of the V chrominance thus appears as in Fig. 7.

The spectrum of the total chrominance signal is the sum of Figs. 6 and 7. Thus the U chrominance appears as bunches separated by integral multiples of line frequency from subcarrier frequency whilst the V chrominance appears as bunches separated by odd multiples of half line-frequency from subcarrier frequency. Therefore the predominant U chrominance energy lies approximately one quarter of line frequency below line harmonics whereas the V chrominance lies above. A typical region is shown in Fig. 8.

For an interlaced system the centres of the U bunches correspond to both zero and maximum vertical spatial frequencies of U. The vertical midband of U corresponds to the region midway between U bunches, that is at the centres of the V bunches. Similarly the centres of the V bunches correspond to both zero and maximum vertical

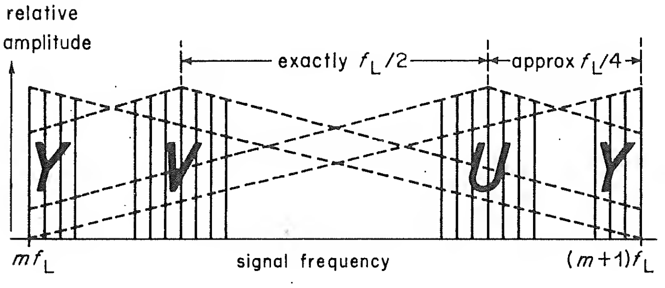


Fig. 8 - Typical region of spectrum near subcarrier frequency

spatial frequenices of V and the vertical midband of V falls at the centres of the U bunches. There is, however, no confusion between U and V. This is because lines within both U and V bunches are separated by the field frequency whereas the separation between U and V bunch centres is exactly half line-frequency. As half line-frequency is not an integral multiple of field frequency, because N is odd, the U and V bunches can overlap and interleave in the same way as the luminance bunches. In fact there can never be any U/V confusion however great the value of n. There can, however, be U/U and V/V confusion as with luminance if n exceeds $N/2$.

Further, there is no Y/U or Y/V confusion. This is because the luminance energy appears at multiples of picture frequency, every other line belonging to the same energy bunch. However, the U energy is spaced at integral multiples of picture frequency from subcarrier frequency, that is at the frequencies.

$$[(q - \tfrac{1}{4})N + r]f_p$$

where q is 284 and r is any integer. As N is divisible by 4 with remainder 1 this expression may be written as

$$(s - \tfrac{1}{4})f_p$$

where s is any integer.

The V energy appears at the frequencies

$$[(q + \tfrac{1}{4})N + r]f_p$$

where r is any integer. This may be written as

$$(s + \tfrac{1}{4})f_p$$

where s is any integer.

A portion of Fig. 8 in the region of a U bunch would therefore appear as in Fig. 9(a). A portion in the region of a V bunch would appear as in Fig. 9(b). Thus the Y, U and V lines never interact in the limit for stationary scenes and it should be possible to separate the Y, U and V components completely.

The spectral structure of the signal is seen to exist at two levels of fineness. On a coarse scale the predominant Y, U and V energy is separated by multiples of $f_L/4$. On a

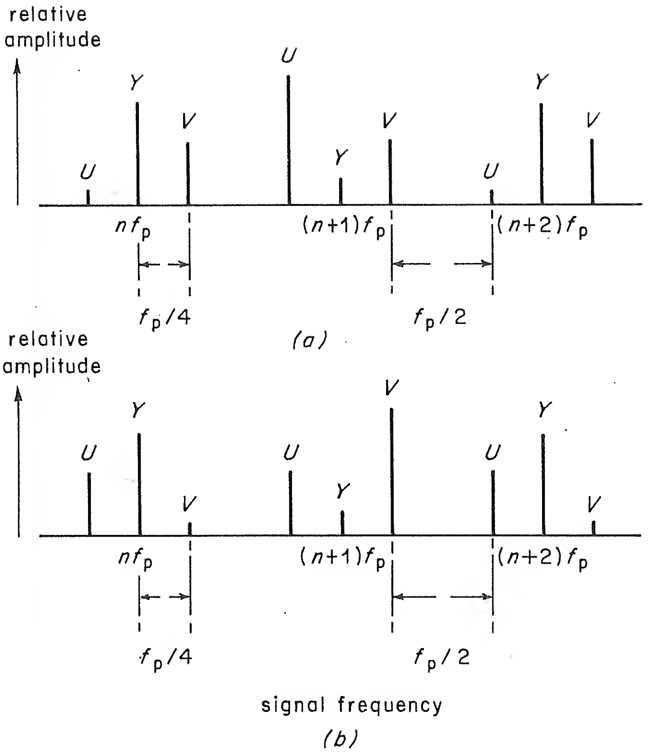


Fig. 9 - Magnified portions of Fig. 8 near the centres of the chrominance spectral bunches
(*a*) near U (*b*) near V

fine scale it is separated by multiples of $f_p/4$. These two facts give rise to filters based on line and field delays.

3.2. Filters based on line delays

3.2.1. Matched filtering

If the filter in Fig. 3 is to pass the predominant chrominance energy then its spectral characteristic must have maxima at odd multiples of $f_L/4$. If, in addition, the system transmits the predominant luminance, then the filter must have minima at integral multiples of f_L so that the luminance is not cancelled there. The simplest idealised characteristic which satisfies these conditions is shown on a linear scale in Fig. 10. This characteristic should only obtain over the modulated chrominance spectral region and elsewhere should be zero.

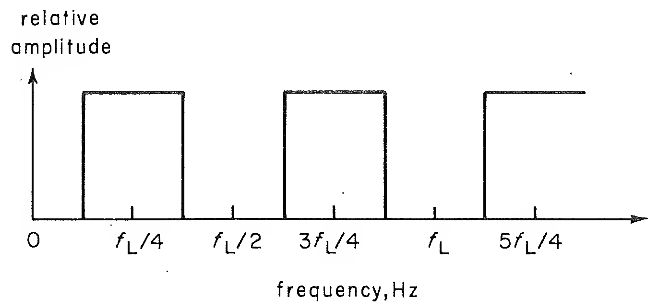


Fig. 10 - Spectral characteristic of ideal filter for separating luminance and chrominance on a line frequency scale

The effect of this filter is to restrict the vertical components of the chrominance spatial frequencies to the ranges $0–{}^1/_8$, ${}^3/_8–{}^5/_8$ and ${}^7/_8–1$ of the theoretical maximum resolution. These ranges correspond to 0–39, 117–195 and 273–312 cycles per picture height. In a delay-line PAL decoder the middle range of vertical frequencies is eliminated by the action of the delay-line which separates U from V. The top range of chrominance frequencies would probably not be resolved by the eye at normal viewing distances. It is thus the bottom range which is of primary interest.

The luminance signal components near subcarrier frequency also have their vertical frequencies restricted to the same ranges. As luminance near subcarrier frequency corresponds to high horizontal frequencies it is certain diagonal spatial frequencies having combinations of high horizontal frequency and certain vertical frequencies which are lost.

The horizontal bandwidth of the chrominance and horizontal range over which diagonal luminance resolution is lost is governed by the frequency range over which the characteristic of Fig. 10 operates. This can be determined using a simple bandpass filter.

The most straightforward way of approaching the characteristic of Fig. 10 is to use a transversal filter. The essential property of such a filter is that its coefficients are the coefficients of the Fourier series describing the spectral characteristic. The idealised square wave of Fig. 10 clearly has an infinite number of Fourier coefficients and therefore requires a transversal filter of infinite extent.

In practice, truncation of the filter will yield a truncated series which approximates to the square wave. The simplest approximation consists of constant and fundamental terms; this may be called a first-order filter. More complex approximations add further odd harmonics; thus only odd-order filters need be considered.

The periodicity of the characteristic indicates that the filter delay elements are multiples of two line periods. Fig. 11 shows the simplest first-order realisation of the filter. Two approaches may be used to determine the coefficient values. Either the spectral characteristic is made maximally flat at multiples of $f_L/4$ with true maxima and minima there. The coefficients are then solutions of simultaneous linear equations. Otherwise the coefficients can be the Fourier coefficients of the square wave in which case the truncated series does not have overall maxima and minima at multiples of $f_L/4$ (except for the first-order case) but gives the minimum mean square deviation from the ideal. In either case the characteristic is anti-symmetrical (except for the constant) about the value of ½.

As shown in Section 2 the matched system, chrominance, luminance and cross-signal characteristics are $F^2(\omega)$, $[1-F(\omega)]^2$ and $F(\omega)\,(1-F(\omega))$. If ω_0 is the frequency of anti-symmetry, then $F(\omega_0+\omega) = 1 - F(\omega_0-\omega)$ and the shapes of the chrominance pass- and stop-bands are identical to those of the luminance. Moreover, the cross-signal characteristics are symmetrical about ω_0.

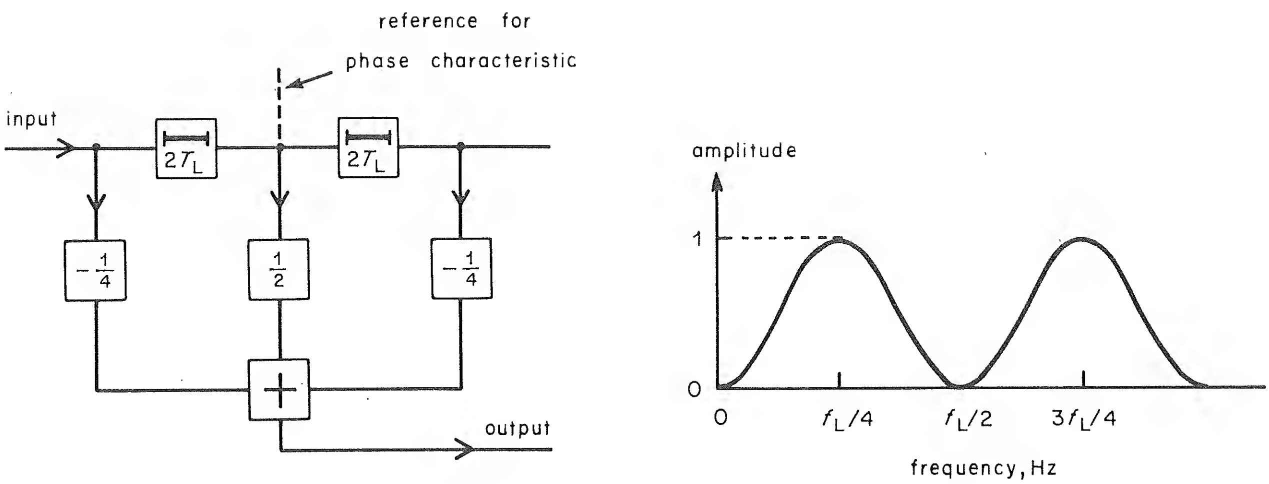


Fig. 11 - The simplest first-order approximation to the characteristic of Fig. 10

(*a*) Filter realisation (*b*) Spectral characteristic

Fig. 12 shows the matched system characteristics of the wanted and cross-signals from the centre of a pass-band to the centre of a stop-band. This range corresponds to a vertical frequency range of 78 cycles per picture height. First-, third- and fifth-order filters are shown in two series. Series (*a*) is the maximally flat case and series (*b*) is the truncated case.

As can be seen, the characteristics of the wanted signal in series (*a*) always steadily decrease from unity (0 dB) so that the cross-signal level rises steadily towards the cross-over point. Thereafter the cross-signal level falls because of the decoder filter characteristic. The infinite suppression at the ends of the range is unnecessary. In series (*b*) this infinite suppression is traded for a sharpening of the peak at the cross-over coupled with a sharper cut of the wanted signal but with pass-band ripple. As the presence of the cross-signal is likely to be more disturbing than the absence of the wanted signal, it is probable that the characteristics of series (*b*) would be preferred.

So far the band-pass filter with which the transversal filter is cascaded has been assumed ideal. In practice its spectral characteristic will have a finite rate of cut and in regions where it is other than unity the characteristics of Fig. 12 will be profoundly modified. Although the effect of the bandpass characteristic on the curves of Fig. 12 can be deduced, it is difficult to appreciate the significance of the result. This treatment is therefore reserved for the two-dimensional approach.

3.2.2. Filtering at only one end of the system

The effect of using a filter at only one end of the system is easiest to appreciate using the ideal characteristic of Fig. 10 in conjunction with Fig. 8.

If the filter is used only at the coder, then the transmitted luminance and chrominance have restricted vertical frequencies. At the decoder, however, no further restriction takes place. Thus all the luminance enters the chrominance channel and all the chrominance enters the luminance channel.

The restriction of luminance frequencies means that the coarse cross-colour having vertical frequencies in the range 0–39 c./p.h. (neglecting the other ranges) is eliminated. But the fine cross-colour in the range 39–117 c./p.h. remains. For example, the fine cross-colour on the vertical gratings of Test Card F is unaffected because the gratings have no vertical frequency components. Nevertheless, it is the coarse cross-colour which is the more objectionable and so the filter gives a worthwhile improvement.

The restriction of chrominance vertical frequencies means that the cross-luminance with vertical frequencies in the ranges 0–39 and 117–195 c./p.h. is eliminated. Because the luminance has a high horizontal frequency component, however, the first range does not appear particularly coarse. More importantly, the chrominance carrier frequencies are not eliminated because, of course, they occur where there is no chrominance detail. Thus plain coloured areas have the full amount of subcarrier patterning which would be completely unacceptable. It is therefore inevitable that some form of conventional notch filter must again be present in the luminance path of the decoder.

If the filter is used only at the decoder then the transmitted luminance and chrominance have unrestricted vertical bandwidths. But at the decoder the separated chrominance has restricted vertical frequencies and cross-colour of the same vertical frequency range. Thus the coarse cross-colour remains but the fine cross-colour is eliminated. For example, the cross-colour on the vertical gratings of Test Card F is now eliminated. Although the fine cross-colour is less objectionable than the coarse it can be argued that it occurs more often because verticals are statistically more probable.

The separated luminance at the decoder also has restricted vertical frequencies and cross-luminance of the same vertical frequency range. Thus the cross-luminance of fine vertical frequency is eliminated. In particular the chrominance carrier frequencies are eliminated so that plain coloured areas are free of subcarrier patterning. Therefore

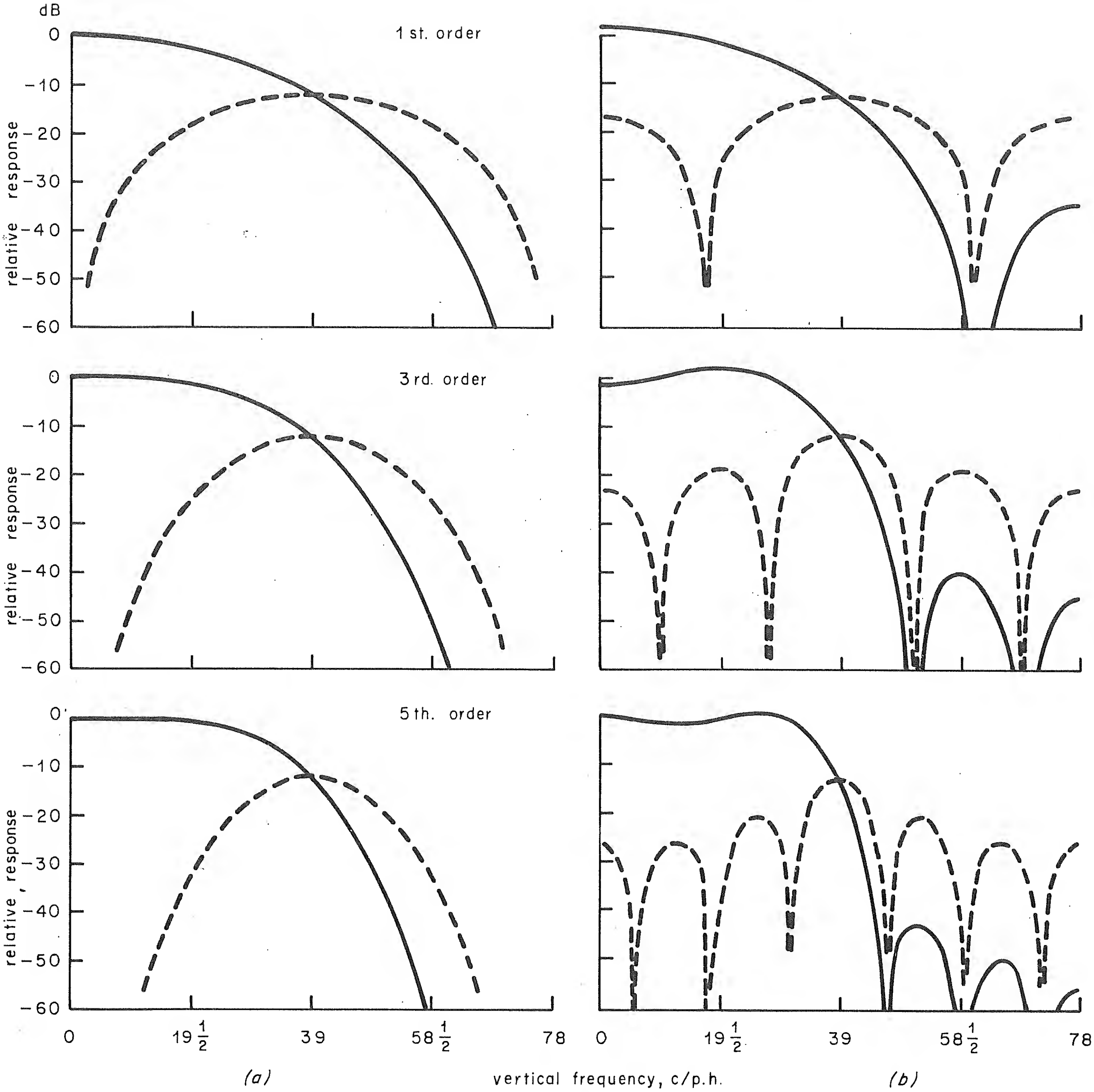


Fig. 12 - Spectral characteristics of a matched system with finite-order filters

(*a*) Maximally flat (*b*) Truncated ——— Wanted signal - - - - - Cross signal

a conventional subcarrier notch filter is unnecessary. All the vertical gratings on the test card are now fully resolved.

The effects of the permutation of filter position are summarised in Table 1.

3.3. Filters based on field delays

Considering the spectral structure of Fig. 9 it is clear that a filter that separates luminance and chrominance on a picture frequency scale must have maxima at odd multiples of $f_p/4$ and minima at integral multiples of f_p. There is also some indication that a field frequency structure is present.

This is about as far as one can go. In particular the effect of the filter on movement is difficult to predict. The treatment of filters based on field delays is therefore reserved for the three-dimensional approach.

4. The two-dimensional approach

4.1. The two-dimensional spectrum

A stationary image is a function of two dimensions and can be represented by a sum of two-dimensional Fourier components. A general Fourier component is a

TABLE 1

		CODER	
		No filtering	Filtering
DECODER	No filtering (other than notch)	1	2
	Filtering (but no notch)	3	4

1. Present situation.

2. Reduction of chrominance vertical resolution to 0–1/8, 3/8–5/8, 7/8–1 of maximum. Luminance diagonals with same vertical frequencies as above lost. Reduction of coarse lower amplitude cross-colour and cross-luminance.

3. Reduction of chrominance vertical resolution and luminance diagonal resolution as above. Reduction of fine, higher amplitude cross-colour and cross-luminance.

4. Reduction of chrominance vertical resolution and luminance diagonal resolution as above. No cross-colour or cross-luminance.

spatial frequency of, say, m cycles per picture width and n cycles per picture height and can be visualised as a sloping grating with a sinusoidal variation of brightness. Just as with one-dimensional frequencies, a two-dimensional frequency can be expressed as a sum of two complex frequencies where m and n can take negative values. The amplitudes of the complex frequencies plotted with m and n as co-ordinates constitute the two-dimensional Fourier spectrum of the image.

The position of any signal frequency in the m-n plane can be found using Equation (2); however only 'picture-locked' frequencies give integral values of m and n. Frequencies in-between have to be represented by the nearest integral values. Using this representation all the arguments of Section 3 can be developed and extended.

For example, vertical aliasing can be explained as follows. For any picture-locked frequency, Equation (2) has an infinite number of solutions, that is, pairs of values of m and n. These pairs of values are separated by $(1, N)$ for Equation (2a) and $(2, N)$ for Equation (2b). This is shown for zero signal frequency in Fig. 13. As zero frequency lies at the centre of the image spectrum the points in Fig. 13 mark the centres of the repeated spectra corresponding to the scanned image. Clearly if the spectra do not overlap their dimension in the n direction must not exceed N, that is, the vertical frequencies in the original spectrum must be restricted to less than $N/2$ cycles/picture height.

This repeating of spectra is fundamental to sampling theory. In one-dimensional terms the spectrum of a sam-

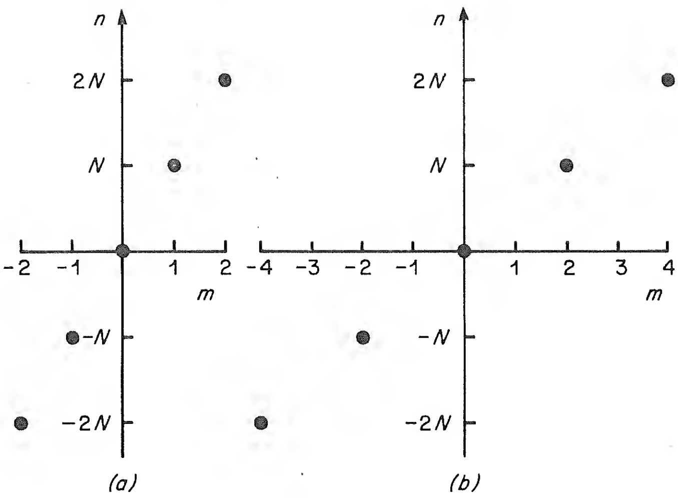


Fig. 13 - The two-dimensional spectral interpretations of zero signal frequency
(a) Sequential system (b) Interlaced system
The horizontal scale is highly magnified

pled function is repeated. In two-dimensional terms the repeat unit has direction as well as magnitude and becomes a vector. The components of the vector are the components of the sampling frequency, in this case $(1, N)$ or $(2, N)$, which describes the scanning action in space.

For an interlaced system the restriction to avoid aliasing is the same as for a sequential system. This is not surprising because a two-dimensional spatial analysis ignores the fact that the fields of a picture occur at different times. But for a filter which operates on the signals of only one field at a time, i.e. intra-field processing, the spatial sampling frequency is twice as coarse and so the spectral characteristic repeats twice as often. Thus the spatial frequency (0, 0) is treated in exactly the same way as the frequency $(1, N/2)$. It will be recalled that in Section 3.1 it was shown that the regions near line-frequency harmonics correspond to both zero and maximum vertical frequencies. Thus a slowly changing filter characteristic, i.e. one with no field-rate detail, treats both zero and maximum vertical frequencies in the same way. This is confirmation of the repeat unit for intra-field processing from the one-dimensional approach.

4.2. The two-dimensional spectrum of the PAL signal

The luminance component is straightforward assuming stationary images. Taking the two interlaced fields together, the spectrum of the luminance image is simply repeated with the unit (2, 625) which is very nearly along the n axis. The upper bound on the signal frequency, ν, placed a bound on the range of values of m and n which can be deduced from Equation (2b). For System I the maximum value of ν is 5·5 MHz and f_L is 15·625 kHz so that the bound is

$$(m - 2n/625) \leqslant 352.$$

For all practical purposes this confines m to less than 352 irrespective of the value of n. As ν is a complex frequency,

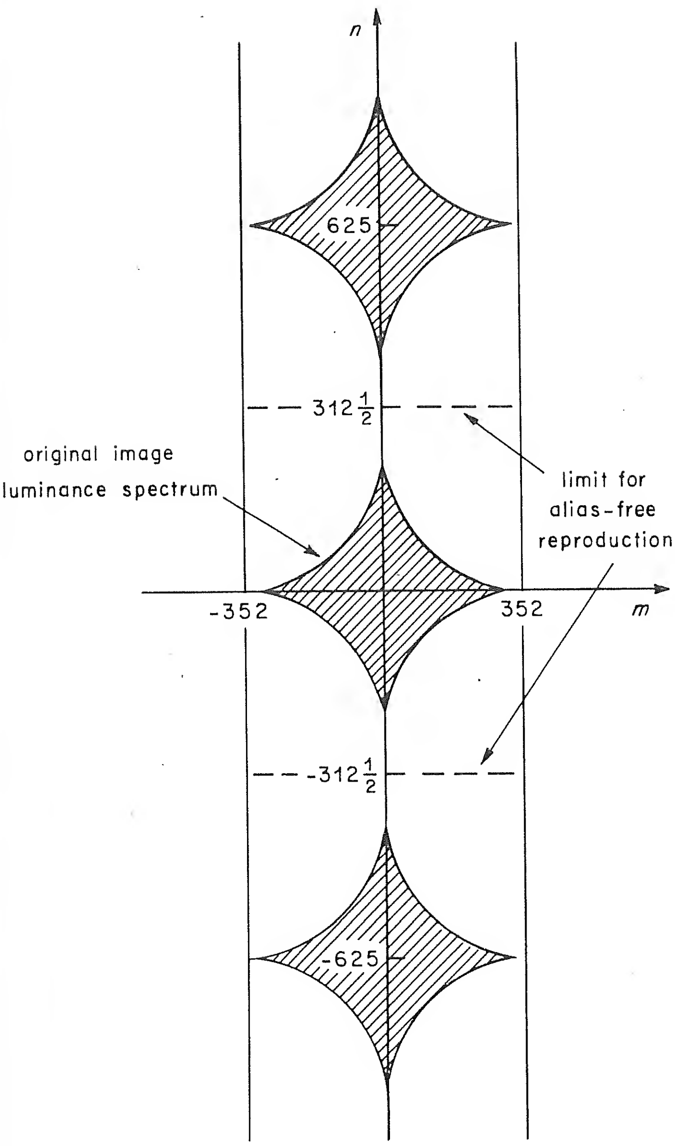


Fig. 14 - The two-dimensional spectrum of the luminance signal

it has a lower bound of −5·5 MHz which imposes a similar lower bound on m. The spectrum of the luminance signal therefore appears as in Fig. 14.

The two figures of ±352 c./p.w. and ±312½ c./p.h. define a rectangle within which the spectrum of alias-free luminance must lie and represent the potentially available spatial resolution of the system. The vertical resolution of 312½ c./p.h. is actually greater in absolute terms than the horizontal by a factor of 1·34 because of the aspect ratio of the full picture.* This fact can be turned to advantage as will be shown later. It also explains why the m and n axes have been differently scaled in Fig. 14.

The chrominance component is more difficult to describe because it is modulated on carriers which are not picture-locked. The spatial spectrum of the chrominance has the same meaning as that of the luminance but the modulation translates the spectrum and centres it on the carriers.

The positions of the carriers can be found using Equation (2b). The U chrominance carrier is subcarrier frequency. The nearest solutions having integral values are (284, 78) and (283, −235). The repeat unit of (2, 625) produces an infinite set of further solutions. For all practical purposes these solutions are parallel to the n axis and separated by 312½ c./p.h.

The V chrominance carrier, as derived in Section 3.1, is a set of carriers of decreasing amplitude, offset from subcarrier frequency by odd multiples of half line-frequency. These frequencies give the nearest solutions (284±r, 234) and (283±r, −79) where r is an integer. Further solutions are provided by the repeat unit as before. For all practical purposes these solutions are also parallel to the n axis and interleave with those for the U chrominance. The positions of the centres of the chrominance regions therefore appear as in Fig. 15.

The restriction of baseband-chrominance signal-frequency imposes a limit on m for the baseband chrominance, as for luminance. Assuming the restriction to be 1·3 MHz, this limits m to 83 c./p.w. After modulation the chrominance regions are therefore limited to 284±83 for the value of m. Taking negative frequencies into account this defines two strips over which chrominance and luminance co-exist, as shown in Fig. 15.

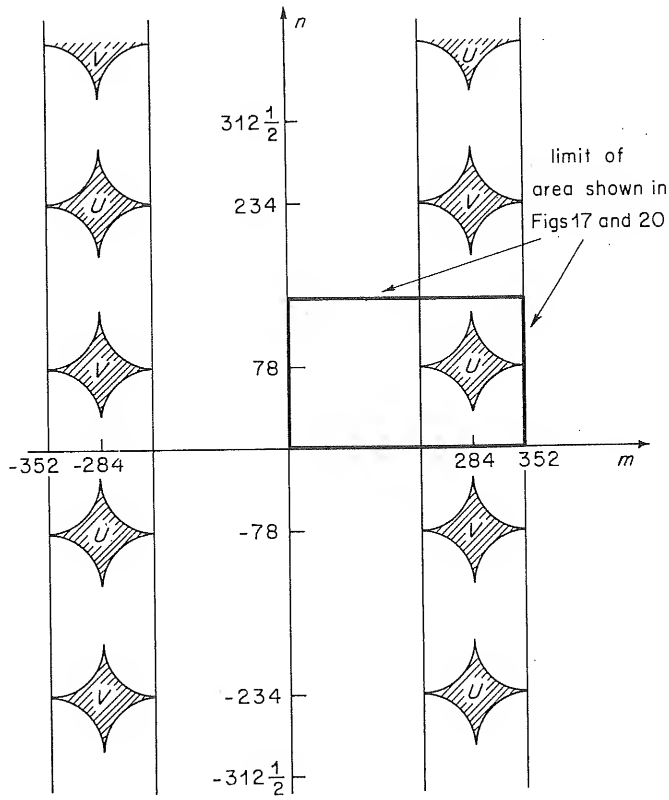


Fig. 15 - The positions of the chrominance regions in the two-dimensional spectrum

* The full picture includes blanking and has an aspect ratio of 1·51.

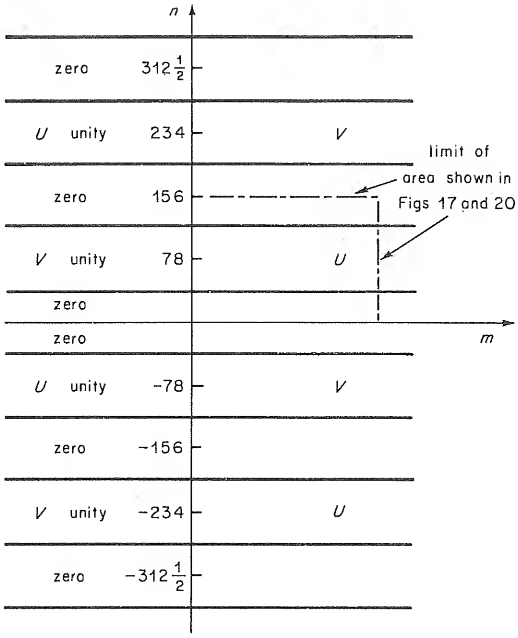


Fig. 16 - Elementary two-dimensional specification for the vertical characteristic of the ideal filter

Using this representation of the PAL signal we can now assess in greater depth the filters developed in Section 3.

4.3. Filters based on line delays

4.3.1. Introduction

The simple bandpass filter clears away all luminance below, say, 3·3 MHz, i.e. for m less than 211, and reserves the remaining space for chrominance. The object of the more complex filters is to reclaim some of the luminance at the expense of the chrominance. This reclaimation should take place where the luminance is most useful and the chrominance least useful.

Statistically the incidence of near vertical edges is higher than that of other inclinations. These edges, having no vertical information, are composed of pure horizontal frequencies which lie along the m axis in the m-n plane. Thus the most useful luminance lies here and the filter characteristic should be zero here. Because of the intra-field repeat unit, the characteristic will then also be zero when n equals any multiple of 312½.

The regions near the chrominance carriers correspond to the coarsest chrominance spatial frequencies that contribute to large areas of colour. At these points the filter characteristic should be unity.

Now, if the chrominance near the m axis is deemed less useful than the luminance, it follows that it may also be eliminated in all the regions midway between the carriers, otherwise the chrominance characteristic will be vertically asymmetrical; thus it is possible to reclaim the diagonal luminance midway between carriers, which has vertical frequencies in a range centred on 156 c./p.h.

Thus an elementary specification for the vertical characteristic of the ideal filter, as shown in Fig. 16, has been derived. This is essentially the same as that of Fig. 10 translated into the m-n plane. In particular, the characteristic repeats vertically with a unit of 156 c./p.h. Thus it is only necessary to describe it over an area bounded by the m and n axes and the lines m = 352 and n = 156 as shown in Fig. 16. (Strictly it need only be described over half this range as it is symmetrical.) The curves of Fig. 12 can now be plotted as contours and the effect of an imperfect bandpass filter can be included.

4.3.2. Variables-separable filters

The filters developed in Section 3.2 take the form of a transversal filter which shapes the vertical frequency response cascaded with a bandpass filter which shapes the horizontal frequency response. Such a behaviour can be termed variables-separable because the shape of the vertical frequency characteristic is simply multiplied by the bandpass characteristic.

The behaviour of the bandpass filter can be quantified by assuming that it, too, is realised in the form of a transversal filter based on element delays. If the element delay corresponds to a sampling frequency of twice subcarrier-frequency and the bandwidth (i.e. the normal modulated chrominance bandwidth) is half the subcarrier-frequency, then the coefficients have easily expressed values as

$$A_{hr} = \tfrac{1}{4}(-1)^r \operatorname{sinc}(r/4)$$

where $\operatorname{sinc} x = (\sin \pi x)/\pi x$.

Fig. 17 shows the overall matched-system chrominance, luminance and cross-signal characteristics expressed in two-dimensional form over the region described in Fig. 16. It will be recalled that these are given by F^2, $(1-F)^2$ and $F(1-F)$ where F is the filter characteristic. Only the first-, third- and fifth-order truncated vertical filters of Fig. 12(b) are shown combined with appropriate truncated horizontal filters. The latter are chosen to give approximately equal rates of cut horizontally and vertically in absolute terms at the band edge.

The corresponding coefficient arrays are shown in Table 2. These are obtained by multiplying together the horizontal and vertical arrays. As the one-dimensional arrays are symmetrical the two-dimensional arrays have four-quadrant symmetry. Therefore only one quadrant is shown. It will be noted that alternate rows are zero because the frequency characteristic repeats vertically every 156 c./p.h.; in addition, alternate remaining rows are zero because there are no even harmonic terms in the spectral characteristic. The boundaries mark the different orders of filters used in Fig. 17.

It will be noted that as the order of the filter rises the area over which the cross-signals are at a level of −12 dB decreases and the characteristics take on an increasingly rectangular shape. This is caused by the variables-separable behaviour of the two cascaded filters. The fact that the

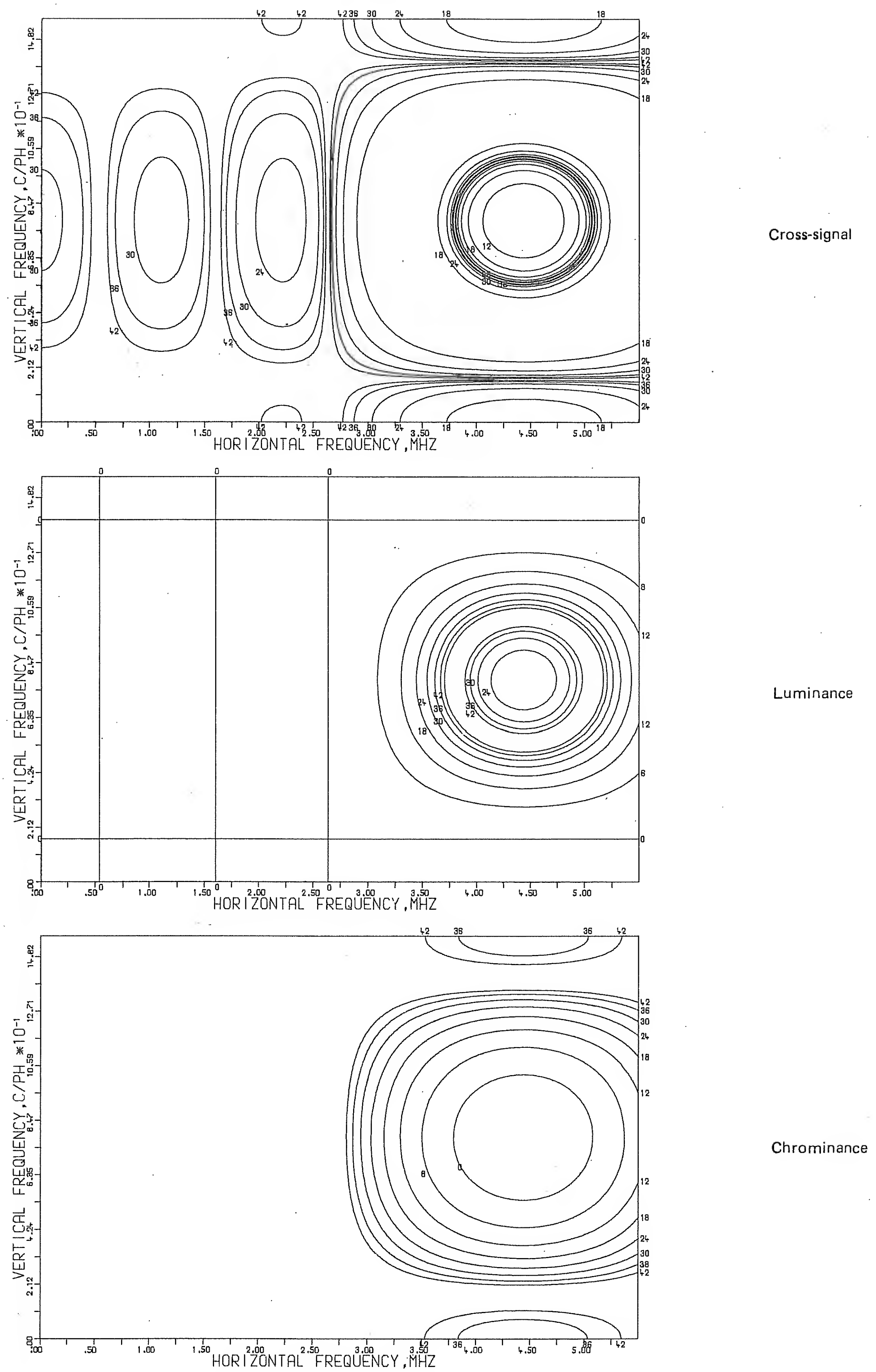
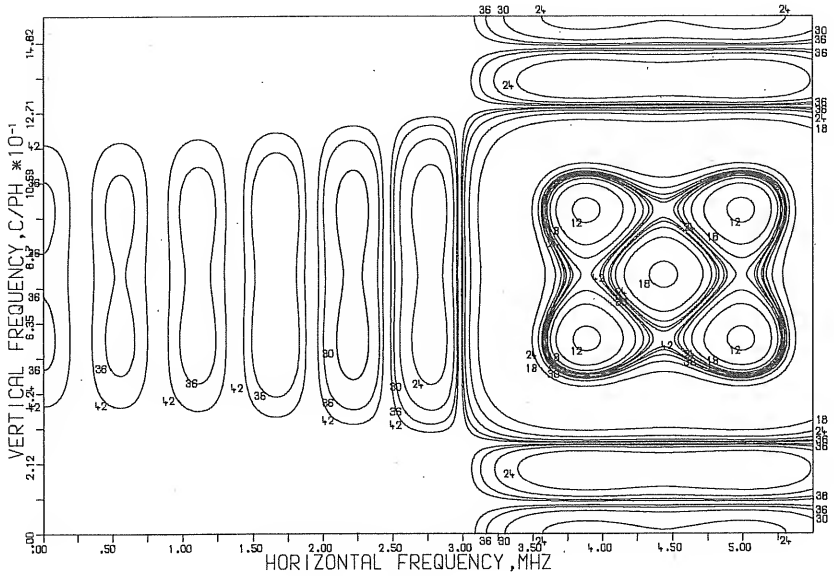
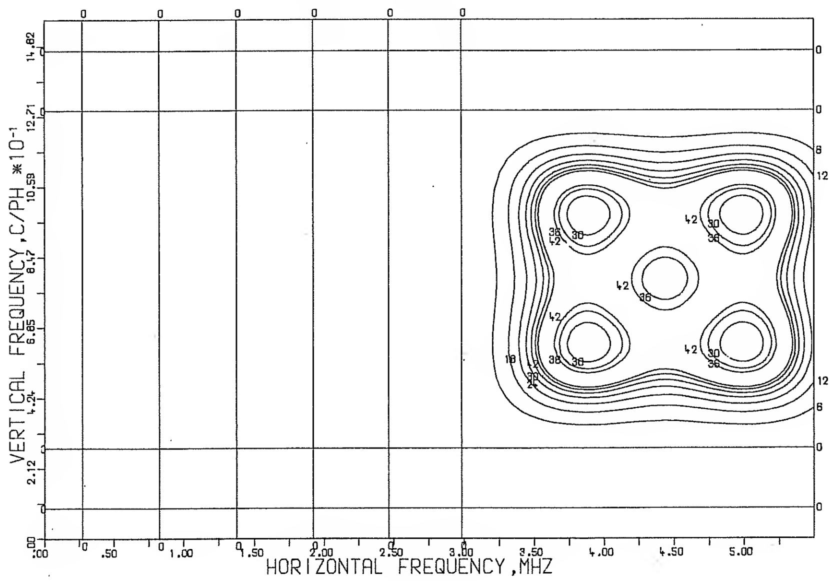


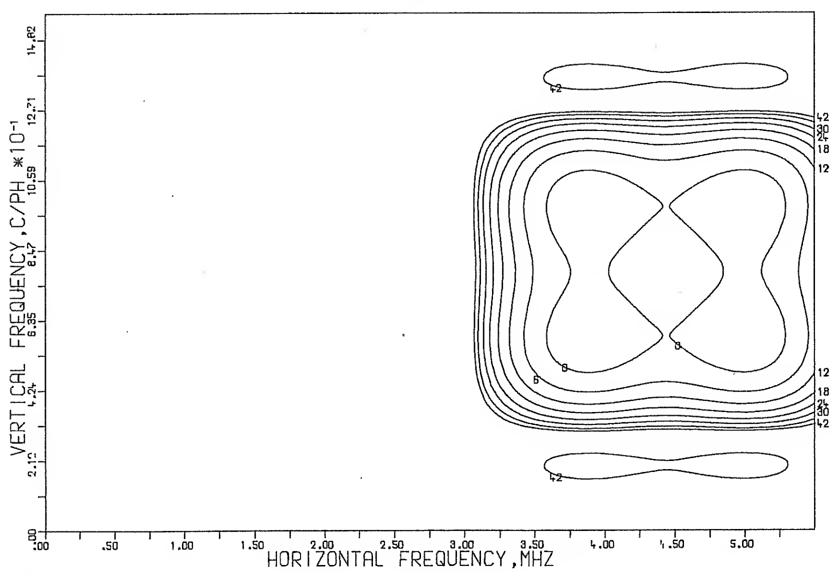
Fig. 17 - Matched system chrominance, luminance and cross spectral characteristics in two-dimensional form
Numbers are dB of attenuation
(*a*) 1st order vertical



Cross-signal

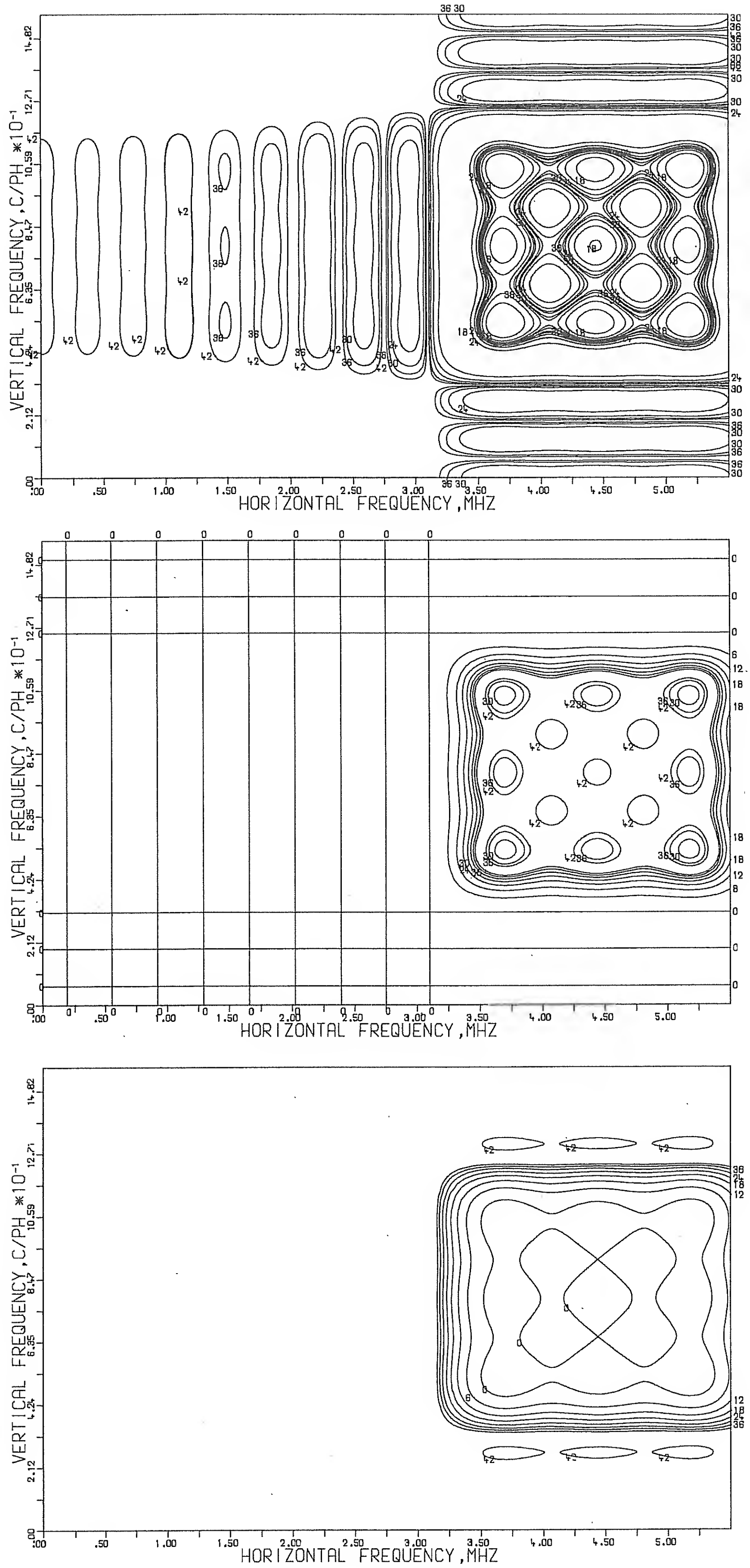


Luminance



Chrominance

Fig. 17 (Cont'd)
(*b*) 3rd order vertical



Cross-signal

Luminance

Chrominance

Fig. 17 (Cont'd)

(*c*) 5th order vertical

TABLE 2

Two-Dimensional Filter Coefficient Arrays

(a) Variables-separable filters

-.0159	.0143	-.0101	.0048	.0000	-.0029	.0034	-.0020	.0000	.0016	-.0020	.0018	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0265	-.0239	.0169	-.0080	.0000	.0048	-.0056	.0034	.0000	-.0027	.0034	-.0022	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.0796	.0716	-.0507	.0239	.0000	-.0143	.0169	-.0102	.0000	.0080	-.0101	.0065	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.1250	-.1125	.0796	-.0375	.0000	.0225	-.0265	.0161	.0000	-.0125	.0159	-.0102	.0000

(a) (b) (c)

The boundaries (a), (b) and (c) refer to the filters of Fig. 17.

TABLE 2

Two-Dimensional Filter Coefficient Arrays

(b) Isotropic filters

-.0054	.0056	-.0063	.0070	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.0098	.0100	-.0104	.0103	-.0089	.0055	-.0005	-.0045	.0072	.0000	.0000	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.0169	.0168	-.0159	.0129	-.0069	-.0013	.0083	-.0104	.0061	.0015	-.0068	.0000	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.0309	.0290	-.0221	.0091	.0063	-.0161	.0148	-.0031	-.0079	.0100	-.0030	-.0053	.0000
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
-.0754	.0552	-.0100	-.0269	.0330	-.0126	-.0105	.0164	-.0053	-.0077	.0096	-.0013	-.0065
.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
.2621	-.2197	.1192	-.0199	-.0308	.0272	.0001	-.0165	.0106	.0043	-.0105	.0040	.0052

(a) (b) (c)

The boundaries (a), (b) and (c) refer to the filters of Fig. 20.

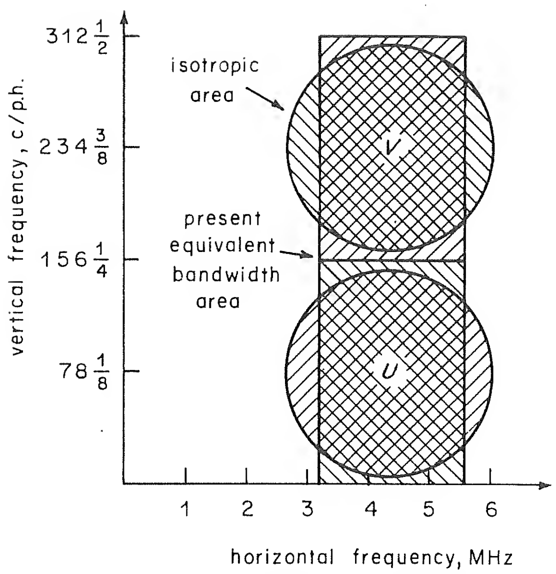


Fig. 18 - The relationship between the present equivalent chrominance spectral area and the equivalent isotropic area

shape is nearly square means that the vertical bandwidth (±1/8 theoretical maximum) corresponds to the chosen horizontal bandwidth of ±¼ subcarrier frequency, i.e. ±1·108. . . MHz. As the latter quantity is considered an acceptable horizontal chrominance bandwidth the filter merely limits the vertical bandwidth to about the same absolute value in spatial-frequency terms.

4.3.3. Isotropic filters

It may be argued that a simultaneous reduction of horizontal and vertical chrominance bandwidth, as described above, would be unacceptable to the eye. In fact subjective evidence of this is lacking. However, the eye's resolution is almost certainly isotropic, i.e. independent of direction, so that it is preferable to limit the horizontal and vertical resolution to the same value.

In a 'delay-line' PAL decoder the vertical chrominance bandwidth is limited by the transversal filter formed by the line delay. This limit amounts to a |cosine| amplitude characteristic having a zero at half the theoretical maximum resolution. The equivalent vertical bandwidth (uniformly filled with the same energy) is ±¼ of the theoretical maximum. As the theoretical maximum corresponds, in absolute terms, to 1·34 x 5·5 MHz horizontally, the equivalent vertical chrominance bandwidth corresponds to about ±1·84 MHz. The horizontal chrominance bandwidth is from Fig. 2 about ±1·2 MHz showing that, with conventional decoders, there is more vertical chrominance resolution than horizontal.

The equivalent isotropic bandwidth, i.e. having the same area in two-dimensional frequency space, is 1·68 MHz horizontally or 0·228 x theoretical maximum vertical resolution. Fig. 18 shows the relationships between the present equivalent bandwidth area and the isotropic area. As can be seen the isotropic area very nearly touches the horizontal axis. If such a filter characteristic were used then lumi-

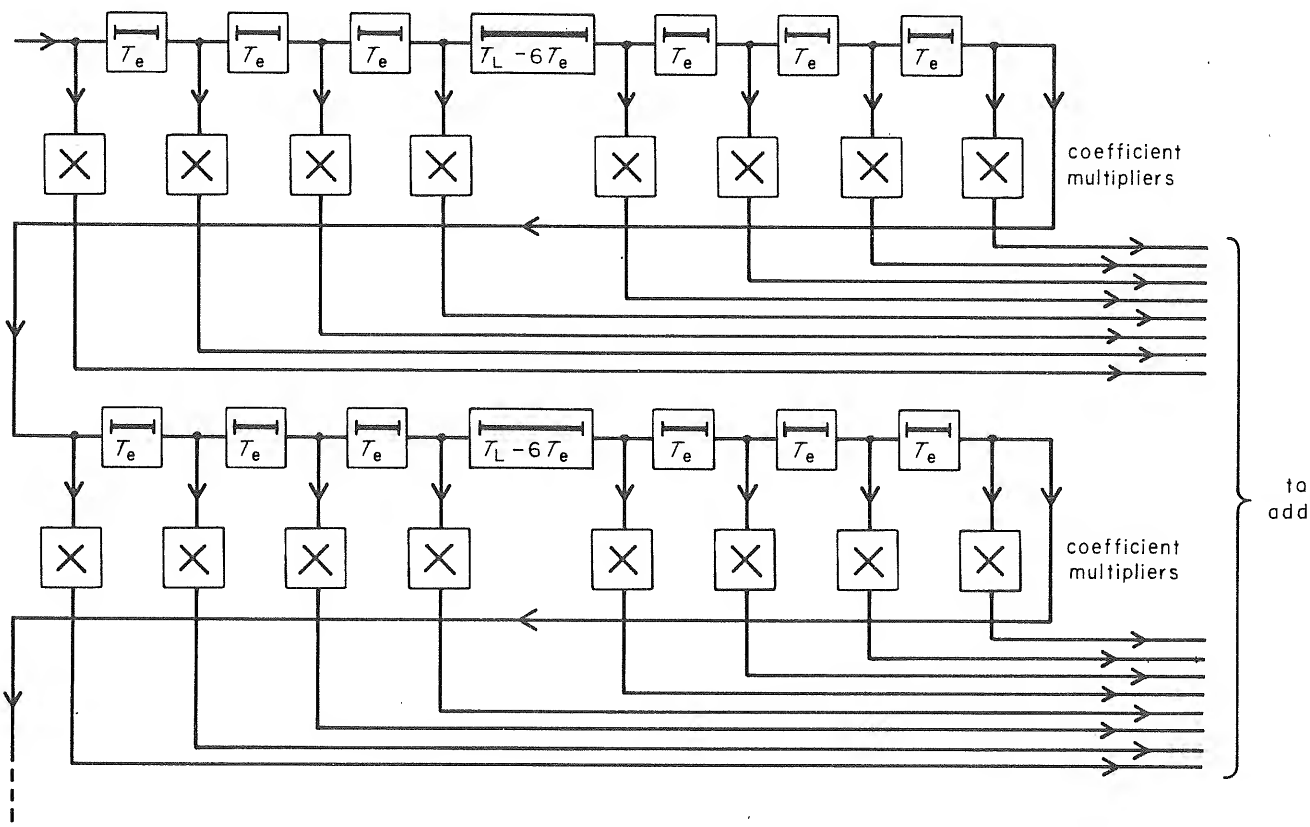


Fig. 19 - One form of generalised two-dimensional transversal filter

nance spatial frequencies near 284 c./p.w. would have to be very nearly vertical to avoid rejection.

It may be that the radius of the isotropic area in Fig. 18 could be reduced without noticeable impairment of the chrominance resolution; subjective tests could confirm this. Again, the degree of exchange of vertical for horizontal bandwidth should be investigated.

An isotropic filter with a characteristic as shown in Fig. 18 is not variables-separable and therefore cannot be realised by cascading horizontal and vertical filters. In one-dimensional terms the width of the spectral teeth is not now constant at $f_L/4$ but must vary from zero to a maximum and back to zero across the chrominance band. The coefficients that weight adjacent picture points are the two-dimensional Fourier coefficients of the frequency characteristic. As this is now not variables-separable, neither is the coefficient array. Thus each picture point must be separately accessible and this requires a generalised form of transversal filter based on shortened two-line delays alternating with groups of element delays as shown in Fig. 19.

An isotropic filter with sharp frequency boundaries would require an infinite coefficient array, as is necessary with variables-separable filters. In practice a finite array gives a finite rate of cut in the frequency domain, as before. However, the shape of the frequency characteristics also depends on the way in which the infinite coefficient array is truncated. An isotropic characteristic requires a coefficient array with polar symmetry so that, ideally, the array should be truncated with a circular boundary. This can only be approximately satisfied with a Cartesian scanning system.

Fig. 20 shows the characteristics obtained by truncating the array with a circular boundary to give approximately the same number of vertical coefficients along the vertical axis as in Fig. 17. This truncation is reflected in the deviation from circularity of the characteristics.

The corresponding arrays are shown in Table 2. Again only one quadrant of the full array is shown. As before, alternate rows are zero but no remaining row is zero.

4.3.4. Summary

Summarising, it has been assumed that the luminance and chrominance vertical bandwidths should be equal and constant over the subcarrier region. This arrangement is easy to realise using cascaded horizontal and vertical filters but might give unacceptable chrominance vertical resolution. To overcome this, the variable-separable behaviour can be abandoned in favour of an isotropic behaviour yielding about the same total chrominance energy, but better distributed. Isotropic filters would, however, be more difficult to realise because independent access to each picture point would be necessary.

5. The three-dimensional approach

5.1. Introduction

The two-dimensional intra-field filters developed in Section 4 solve the problems of cross-colour and cross-luminance at the expense of severely limiting the chrominance vertical resolution. Isotropic filters may alleviate the problem somewhat, but there is still a serious degradation of resolution. On the other hand, there is a hint in Section 3 that chrominance and luminance can be separated perfectly without any degradation for stationary images. However, this could be done only at the expense of degrading moving images. It is the purpose of this section to examine the nature of this degradation and develop filters that would produce minimal impairment. For this purpose we need to develop the concept of the three-dimensional spectrum.

5.2. The three-dimensional spectrum

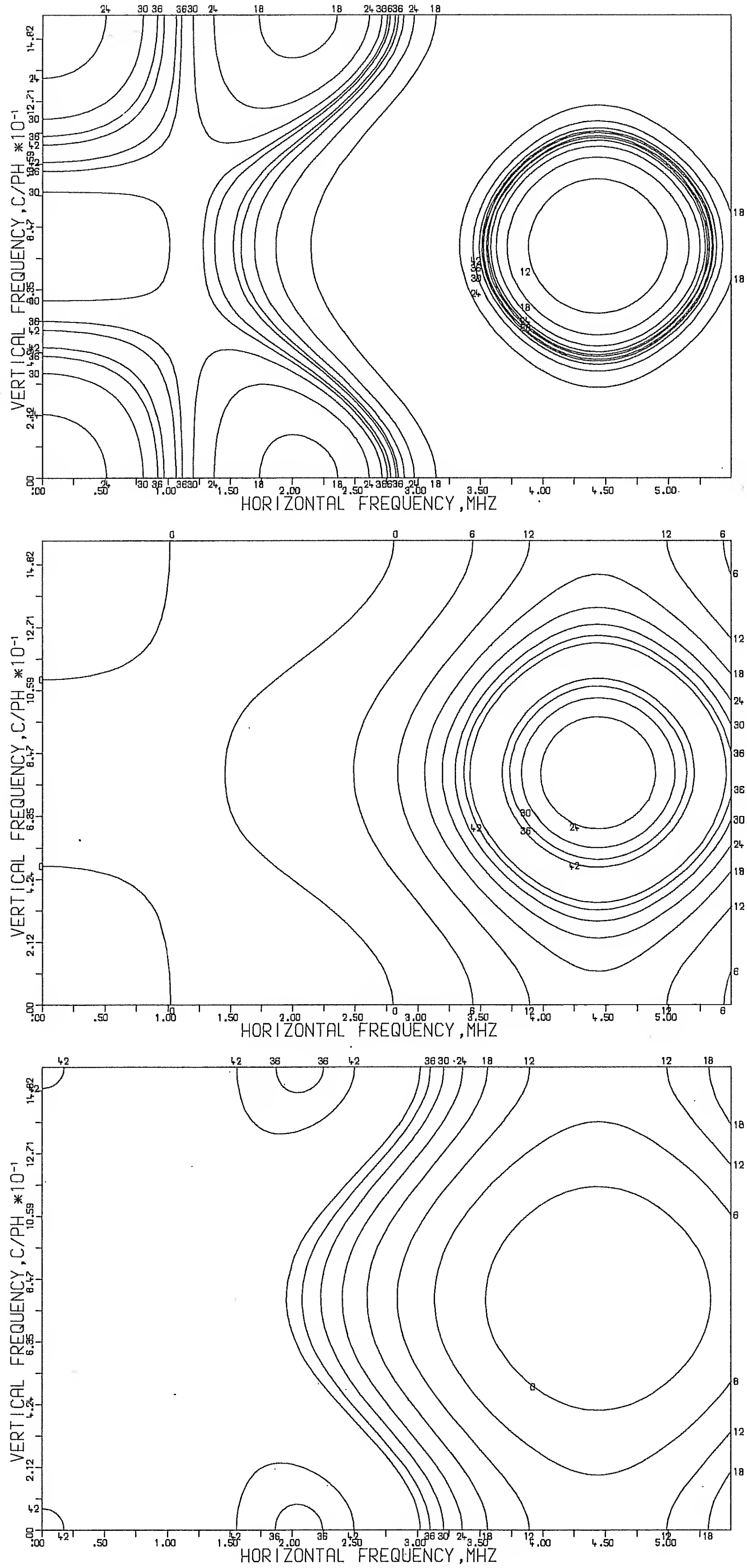
A moving image can be described as a summation of 'moving' spatial frequencies. A general moving spatial frequency has, say, m cycles per picture width and n cycles per picture height and can be visualised as moving at a speed such that crests pass a fixed point at f Hz; it can be regarded as an infinite moving grating where the direction of motion is indeterminate (only the sense is determinate). It can be expressed in terms of complex frequencies where m, n and f can be negative and the amplitudes (and phases) of the frequencies plotted with m, n and f as co-ordinates constitute the three-dimensional spectrum.

The position of any arbitrary signal frequency in the spectral space can be determined through Equation (2) extended to include movement. For interlaced scanning this becomes

$$\nu = (m - 2n/N)f_L + f \qquad (3)$$

where f is positive if the grating appears to move to the left. The inclusion of the extra term means that any signal frequency can be given exact co-ordinates where m and n are integers, but f not necessarily so. The difficulties met in Section 4 can now be resolved.

Just as with two-dimensional space, Equation (3) has an infinite number of solutions corresponding to any value of ν. This means that spectra are repeated but now the repeat unit is a three-dimensional vector. In fact, there are two such vectors whose components are those of the three-dimensional sampling frequencies describing the scanning action. In two dimensions the scanning frequency was a set of lines described by the co-ordinates (2, 625). In three dimensions the scanning frequency is a set of planes or, more correctly, two sets whose intersection describes the scanning action. One set has co-ordinates (0, 1, 50) with planes that are very nearly perpendicular to the time axis. The other set has co-ordinates (1, 313, 25) whose planes intersect the first set to give 312½ lines per picture height, vertically staggered on successive fields according to the interlace pattern. The repetition of spectra based on these two vectors gives a plane lattice in the three-dimensional



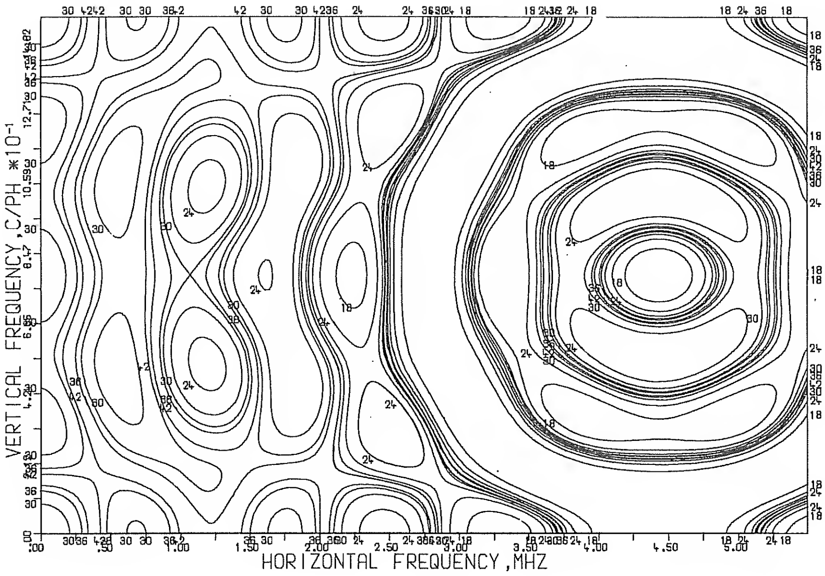
Cross-signal

Luminance

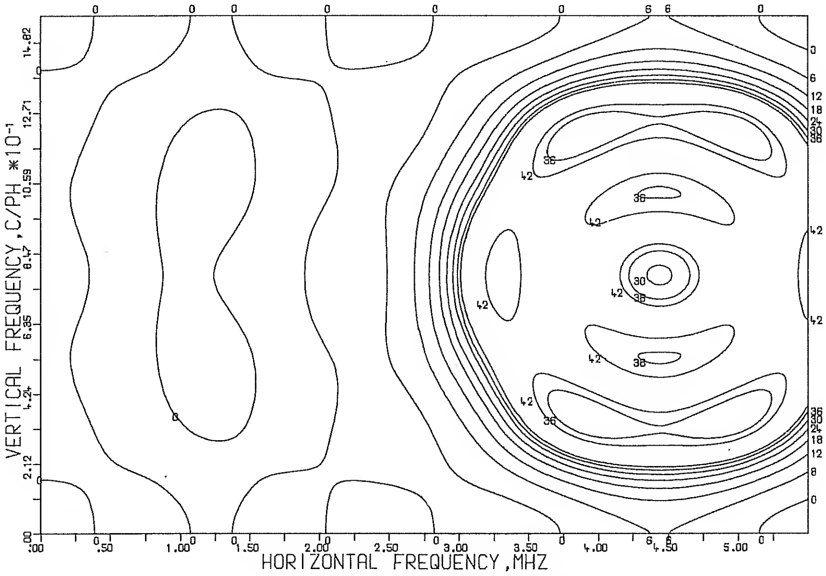
Chrominance

Fig. 20 - Matched system spectral characteristics in two-dimensions. Isotropic filters

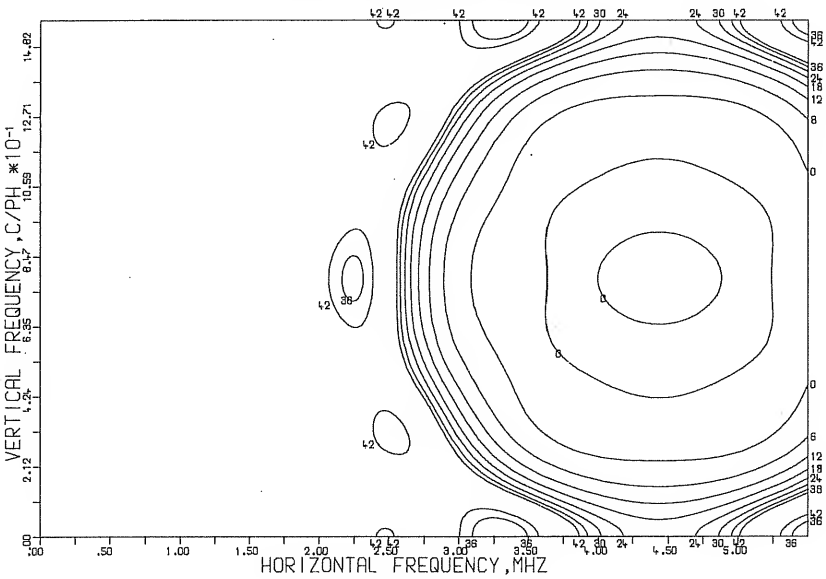
(a) 1st order vertical equivalent



Cross-signal



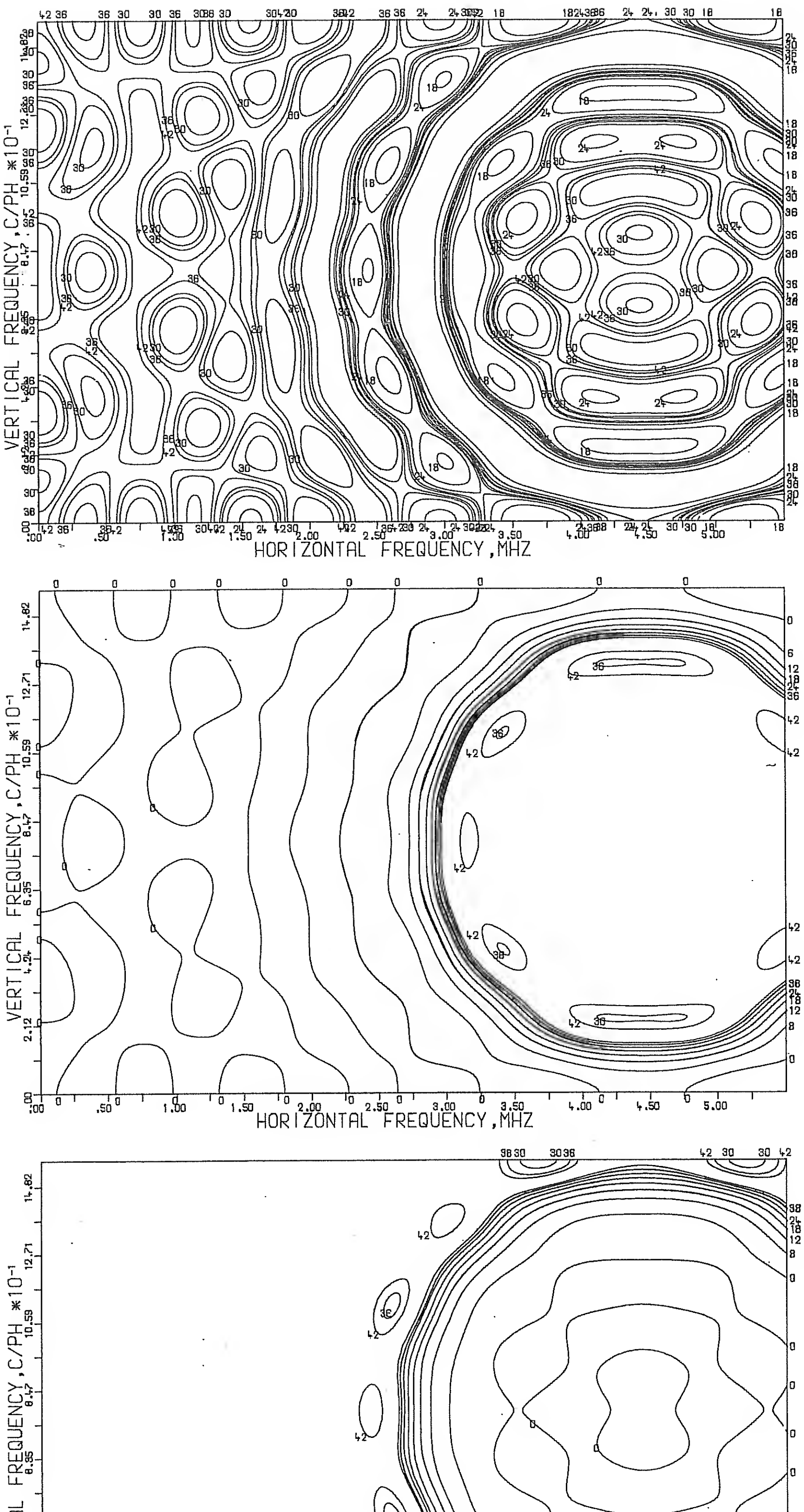
Luminance



Chrominance

Fig. 20 (Cont'd)

(*b*) 3rd order vertical equivalent



Cross-signal

Luminance

Chrominance

Fig. 20 (Cont'd)

(*c*) 5th order vertical equivalent

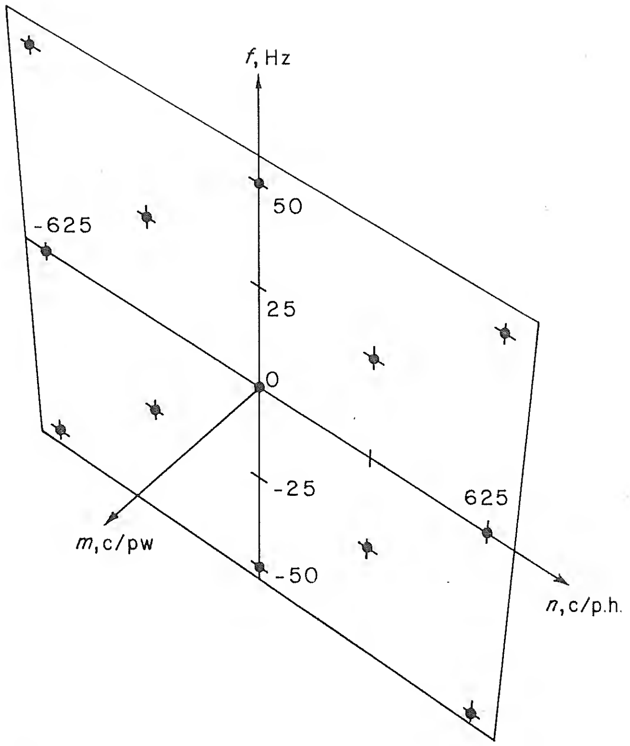


Fig. 21 - The three-dimensional spectral interpretations of zero signal frequency for interlaced scanning

space which is very nearly parallel to the n-f plane. This is shown for zero signal frequency in Fig. 21 where the points mark the centres of the repeated spectra.

The pattern of Fig. 21 is consistent with the two-dimensional approach. For example, taking the two interlaced fields together is tantamount to excluding all temporal variations. This in turn excludes the existence of spectra outside the m-n plane. If the spectra (0, 1, 50), (1, 313, 25), etc., are excluded, the lattice collapses to a one-dimensional form which agrees with that developed in Section 4. Again, if temporal variations are allowed, the spectra (1, 313, 25) and (1, 312, −25) cancel out to a first order because their sum represents a fine spatial frequency vibrating at 25 Hz.

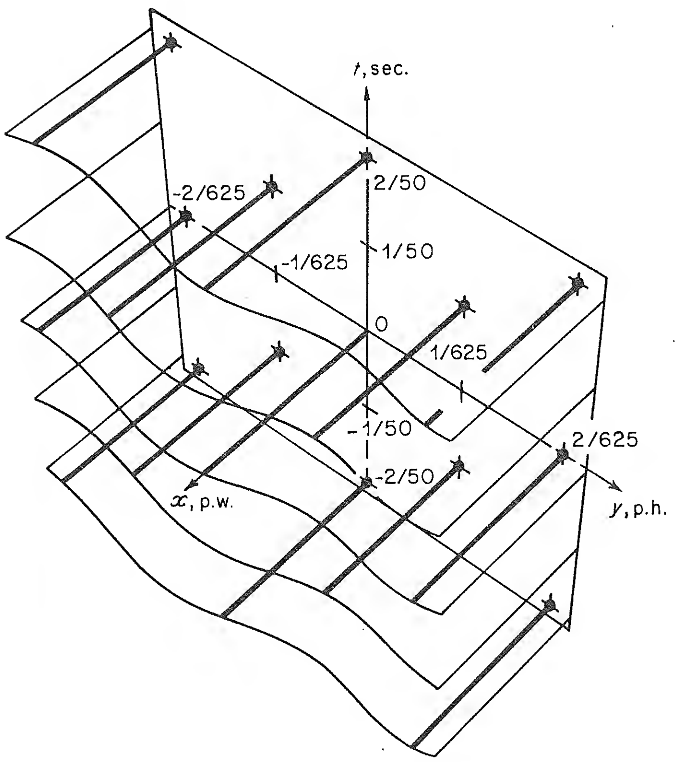


Fig. 22 - The sampling lattice of interlaced scanning

The spatio-temporal sampling lattice of interlace which gives rise to the spectral lattice is shown in Fig. 22. As can be seen, the parameters in x, y, t space are simply the reciprocals of those in the m, n, f space. The two lattices can thus be regarded as reciprocals of each other.

Any filter based on the sampling lattice of Fig. 22 will have a spectral characteristic which repeats according to the pattern of Fig. 21. A transversal filter can be described in terms of its coefficients, which weight adjacent samples. These coefficients, arranged in the array of Fig. 22, are the Fourier coefficients of the two-dimensional series representing the spectral characteristic which repeats with the pattern of Fig. 21. It should therefore be clear that a purely intra-field filter which takes contributions from points in only one field has a spectral characteristic with no f variation. According to Fig. 21 it must therefore repeat in the m-n plane with half the unit of the spectral lattice. This confirms the observations of previous sections.

5.3. The interpretation of movement in the space

When an image moves (more specifically translates) the spatial frequencies which constitute it must move correspondingly. Now it was pointed out in Section 5.2 that the motion of an infinite two-dimensional grating is indeterminate. Only the rate at which crests pass a fixed point can be specified. If this can be found, then the f component is found for any spatial frequency (m, n) for any rate of movement.

To find the rate at which crests pass a fixed point it is simpler to consider a point moving over a fixed grating. Suppose the grating is described by the expression

$$\cos 2\pi(mx/2a + ny/2b)$$

where $2a$ and $2b$ are the picture width and height respectively. Suppose the motion of the point is described by

$$x = ut, \qquad y = vt$$

where u and v are the velocity components. Then substituting for x and y, we find that the disturbance at the point is given by

$$\cos 2\pi(mu/2a + nv/2b)t.$$

Now this describes a temporal variation of frequency f where f is given by

$$f = m(u/2a) + n(v/2b)$$

If the velocity components of the point relative to the grating are u and v, it follows that the velocity components of the grating relative to the point are $-u$ and $-v$. Thus the f component for the moving grating is given by

$$f = -m(u/2a) - n(v/2b) \tag{4}$$

The quantities $(u/2a)$ and $(v/2b)$ are simply the grating velocity components expressed in picture widths and picture heights per second. Provided that neither m nor n is zero, there are an infinite number of combinations of these components which give the same value of f. This explains why the grating velocity is indeterminate.

Equation (4) describes a plane in the m, n, f space. It passes through the origin and has a gradient of $[(u/2a)^2 + (v/2b)^2]^{1/2}$ which is the magnitude of the velocity expressed in picture dimensions. The angle of the line of steepest descent with respect to the m axis is $\tan^{-1}(av/bu)$ which is the direction of the velocity, again modified by the picture dimensions.

This enables an overall picture of the spectrum of an image to be gained. When stationary it lies wholly in the m-n plane. When it moves it lies on a tilted plane whose inclination is proportional to the velocity and whose direction of tilt is in the direction of motion.

5.4. The three-dimensional spectrum of the PAL signal

The arguments are similar to those of Section 4.2 except that areas become volumes. The image luminance-spectrum, now three-dimensional, is repeated, centred on the points of Fig. 21. This creates a layer nearly parallel to the n-f plane as shown in Fig. 23. As before, $|m|$ is limited to 352 which limits the layer thickness.

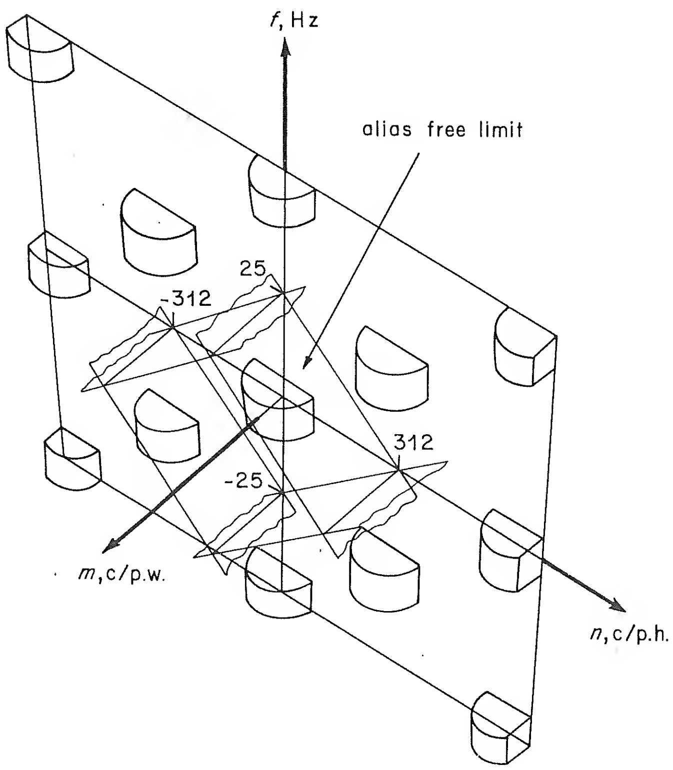


Fig. 23 - The three-dimensional spectrum of the luminance signal

Fig. 23 implies that the alias-free luminance volume resembles a rhombic prism because the cross-section in the n-f plane is a rhombus. The cross-section in the m-n plane is a rectangle as observed in Section 4.2 and there is an inequality between the m and n components. All pure horizontal spatial frequencies, having $n = 0$, may move at up to 25 Hz; but for pure vertical frequencies there is a trade-off between spatial and temporal components. Thus coarse vertical frequencies may move at nearly 25 Hz but fine ones can hardly move at all.

This restriction on movement of spatial frequencies for alias-free reproduction can be represented in another way. Instead of describing the limiting temporal frequency for each spatial frequency it is possible to derive the equivalent spatial bandwidth for the moving object; this concept is sometimes referred to as dynamic resolution. To derive the equivalent it is necessary to use the result proved above, that the spectrum of a moving object lies in a tilted plane and to find where the tilted plane cuts the limiting surface. The projection of the intersection on the m-n plane then gives the spatial bandwidth. This is shown in Fig. 24 for pure horizontal, pure vertical and diagonal movement. It can be seen that, for alias-free reproduction, horizontal and vertical movement are treated very differently.

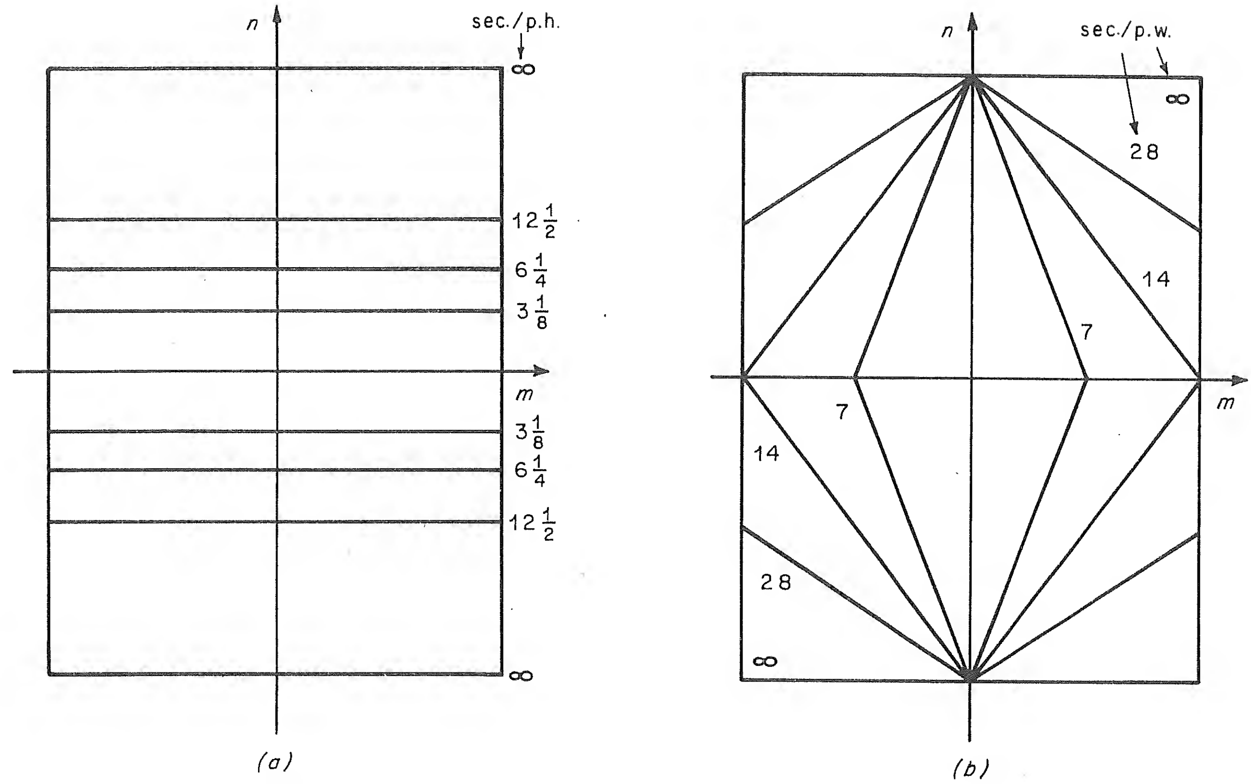
Using the concept of dynamic spatial resolution it is possible to describe the action of any three-dimensional filter in a more tangible way.

As in Section 4.2 the chrominance components of the PAL signal are centred on their respective carriers and so the positions of these carriers in the three-dimensional space must be determined.

Fig. 25(a) shows the principal interpretations of sub-carrier frequency having the lowest values of n and f. These points mark the centres of the regions of U chrominance and their co-ordinates are (284, 78, 18¾) and (283, −235, −6¼); also shown are the positions of the principal interpretations of the V chrominance carrier. It will be recalled that these are really a series of carriers whose co-ordinates are (283$\pm r$, −79, −18¾) and (284$\pm r$, 234, 6¼).

For clarity a section of Fig. 25(a) at a value of m equal to 284 is shown in Fig. 25(b).

It will be observed in Fig. 25(b) that the carrier positions are well separated from the m axis. Thus the carrier of coarser vertical frequency has the finer temporal frequency. This is assured by the 25 Hz offset of the sub-carrier frequency from quarter line-frequency. If it were not so, the temporal frequencies of the carriers would be interchanged and those of coarser vertical frequency would also be those of coarser temporal frequency. This would be undesirable from the point of view of the visibility of cross-colour and cross-luminance.



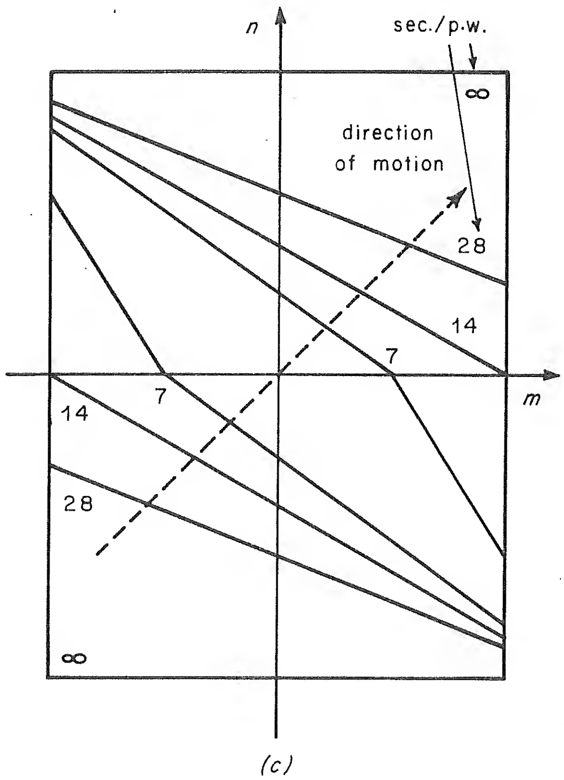


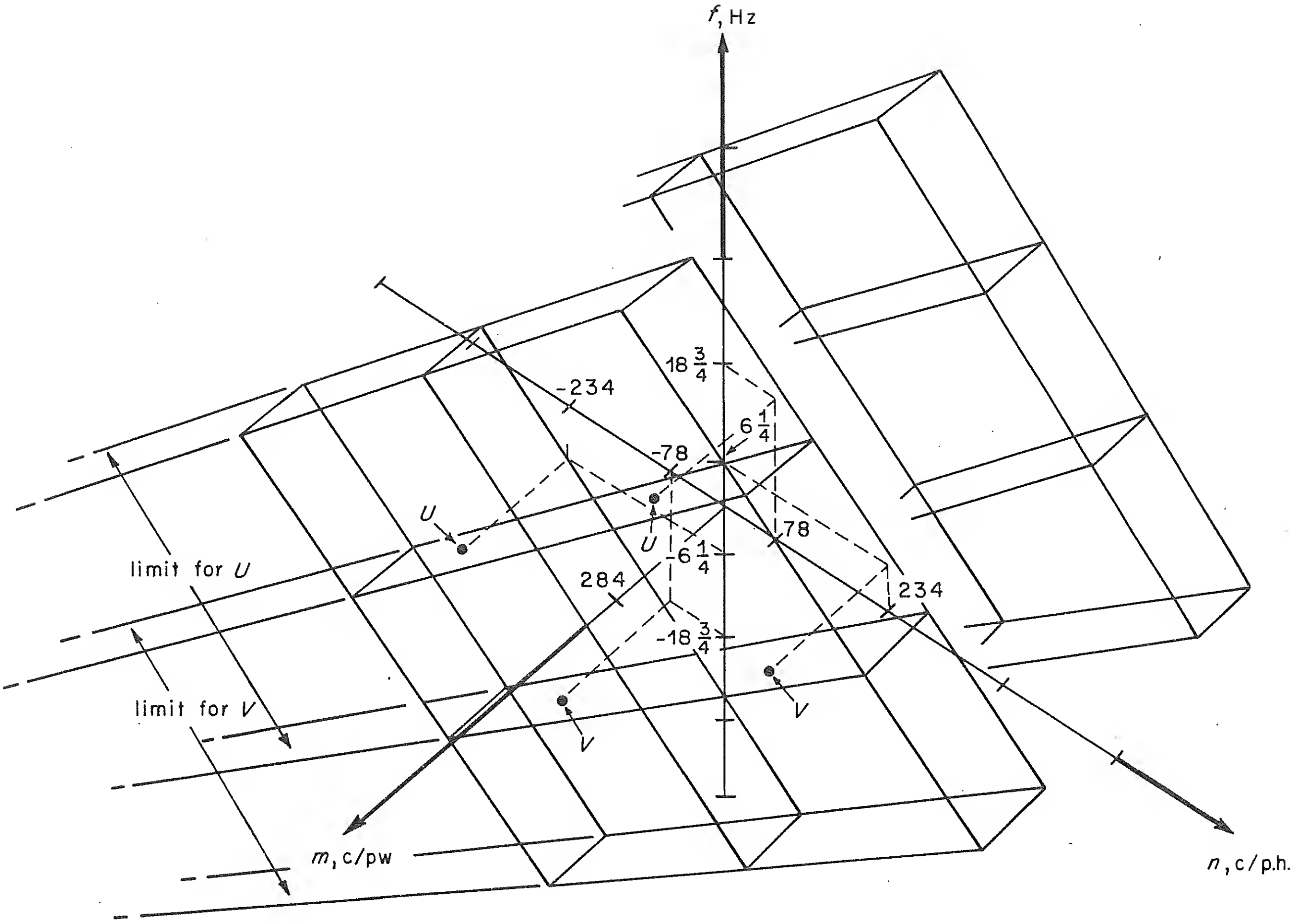
Fig. 24 - The luminance dynamic resolution for alias-free reproduction

(*a*) Vertical motion (*b*) Horizontal motion
(*c*) 45° diagonal motion

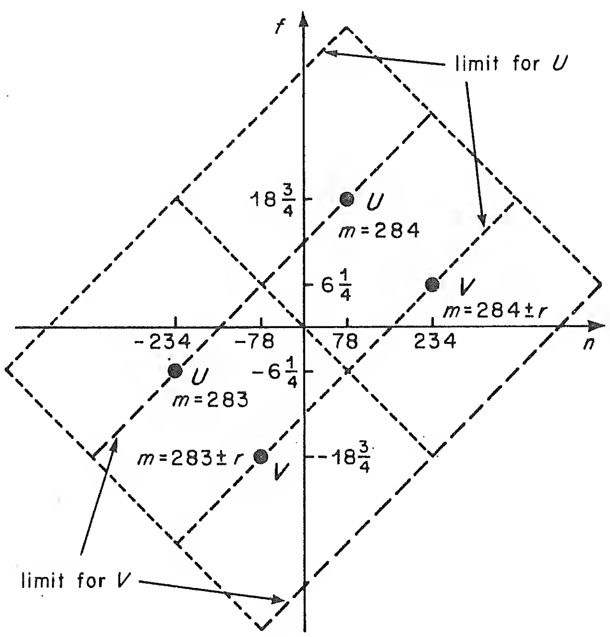
Fig. 25 also shows the alias-free limit of chrominance frequencies that can be handled by the system. Although this limit avoids U/U or V/V confusion, it does not take cross-colour into account; to avoid this, further limitation is necessary as will be seen. The alias-free n-f cross-section is the same as for the luminance, but the limiting of $|m|$ to, say, 83, means that the alias-free volume for the chrominance is a somewhat thinner rhombic prism. The dynamic resolution is thus the same as in Fig. 24 except that the horizontal resolution is cut off at a lower limit.

5.5. Intra-field filters

Using the representation of Fig. 25 it is possible to



(a)



(b)

Fig. 25 - The positions of the chrominance regions in the three-dimensional spectrum

(a) perspective view (b) projection perpendicular to m axis

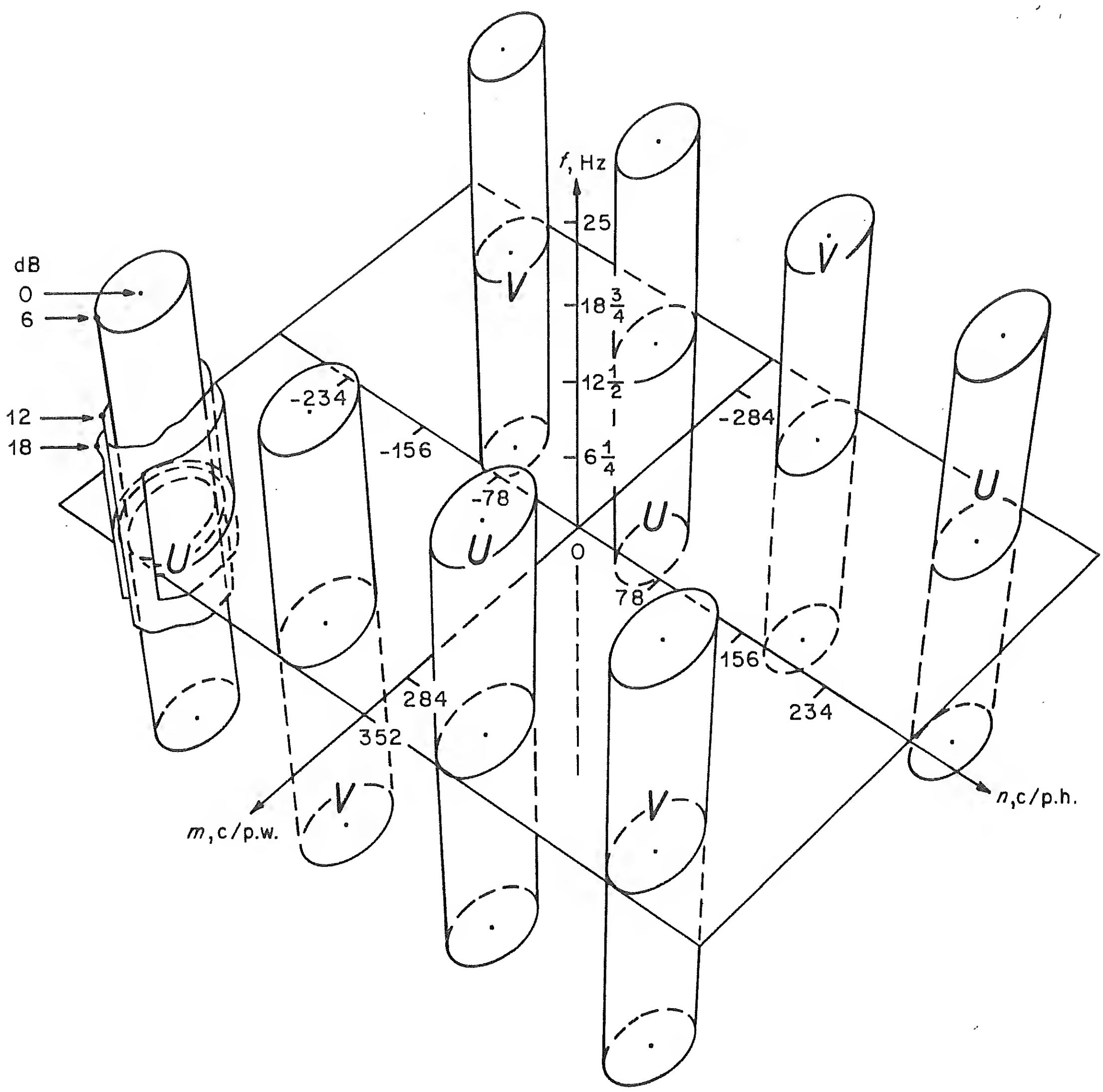


Fig. 26 - The general three-dimensional spectral characteristic of the intra-field filters

consider the effect of the filters developed in Section 4. Fig. 26 shows the general characteristic of these filters. The diagram shows that because no variation along the f axis is possible the filter fares badly when trying to select the regions near the chrominance carriers. Its indiscriminate action also selects the stationary luminance with the same spatial co-ordinates as the carriers so that the luminance characteristic (which is the complement) has no components in these areas.

However, the diagram also points the way towards the kind of filter characteristic that would overcome the problem; Fig. 27 shows an outline of such a characteristic. As can be seen, it varies along the f axis and thus the filter cannot be purely of an intra-field nature. Before it is possible to define the areas of the maxima and minima, however, it is necessary to know whether such a characteristic is reasonable, and this can be attempted by making assumptions about the eye's characteristics and finding the corresponding representation of the idealised PAL signal.

5.6. A model for the idealised PAL signal

Suppose, firstly, that it is possible to exchange spatial and temporal resolution. This is a reasonable assumption based on known properties of the eye.[3,4,5] A convenient relationship would be a linear decrease of temporal resolution with increasing spatial resolution because this is implicit in the way interlace is assumed to work. As the eye's spatial resolution is very nearly isotropic, the eye's characteristic in three-dimensional frequency space can be assumed to be a surface of the form shown in Fig. 28; i.e. it can be represented by two cones, base to base. In reality, the eye's characteristic is exceedingly complex, but the surface of Fig. 28 is a good approximation to a typical surface of constant spatio-temporal response.

Suppose, secondly, that this surface represents the characteristic of the eye's response to both luminance and chrominance. However, the relative dimensions would be different in each case; for example, it is known that the

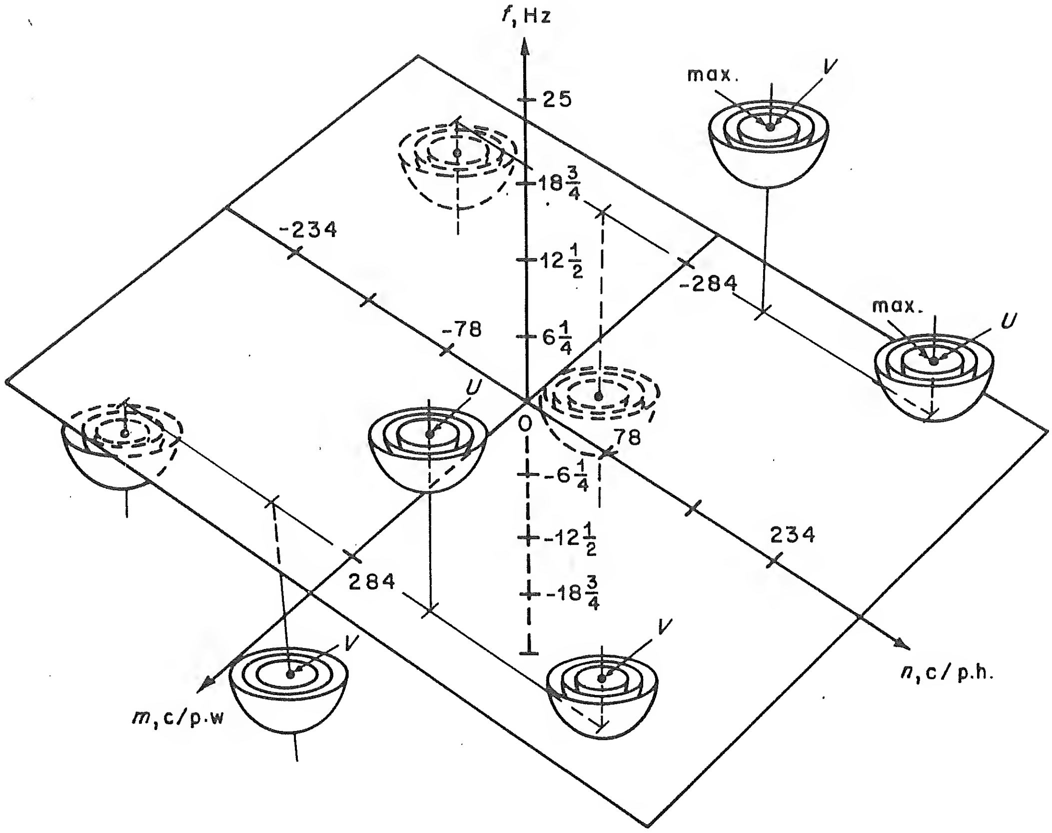


Fig. 27 - An outline of the required three-dimensional filter characteristic

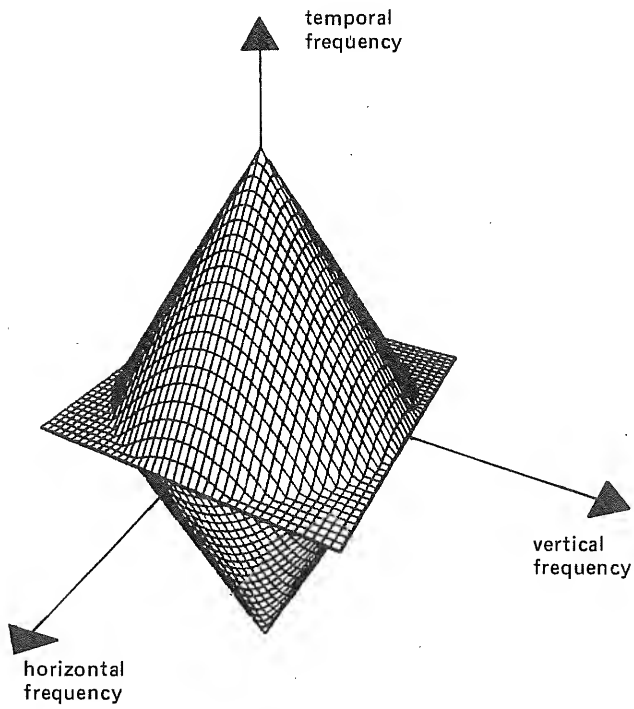


Fig. 28 - A typical surface representing the locus of constant spatio-temporal response of the eye

eye's spatial resolution for chrominance is much less than that for luminance.

Suppose, then, that the spectrum of the original scene is filtered so that all components beyond the bounds of the surface are eliminated. Then the spectrum of the PAL signal would be bounded by surfaces like that of Fig. 28, one centred on zero frequency for the luminance and one each centred on the chrominance carriers. A perspective view is shown in Fig. 29. Moreover, because of the scanning action, the whole structure would repeat according to the pattern of Fig. 21.

It now only remains to specify reasonable dimensions for the surfaces. It will be recalled that for System I there is potentially more vertical resolution than horizontal. This is not accidental for it has long been known that the full vertical potential cannot be realised due to display imperfections.* It would seem reasonable, therefore, to limit the required luminance vertical-resolution to the same value, in absolute terms, as the horizontal resolution. Thus the luminance cone radius corresponds to 5·5 MHz or 234 c./p.h. The maximum temporal resolution, that is, the cone height, can be 25 Hz without aliasing.

The chrominance spatial resolution is more open to question. For example, the equivalent present isotropic radius of 1·68 MHz derived in Section 4.3.3 could be taken. For simplicity, the figure of 1·84 MHz will be assumed;

* A display with a 'box-car' persistence characteristic would be free from these imperfections.

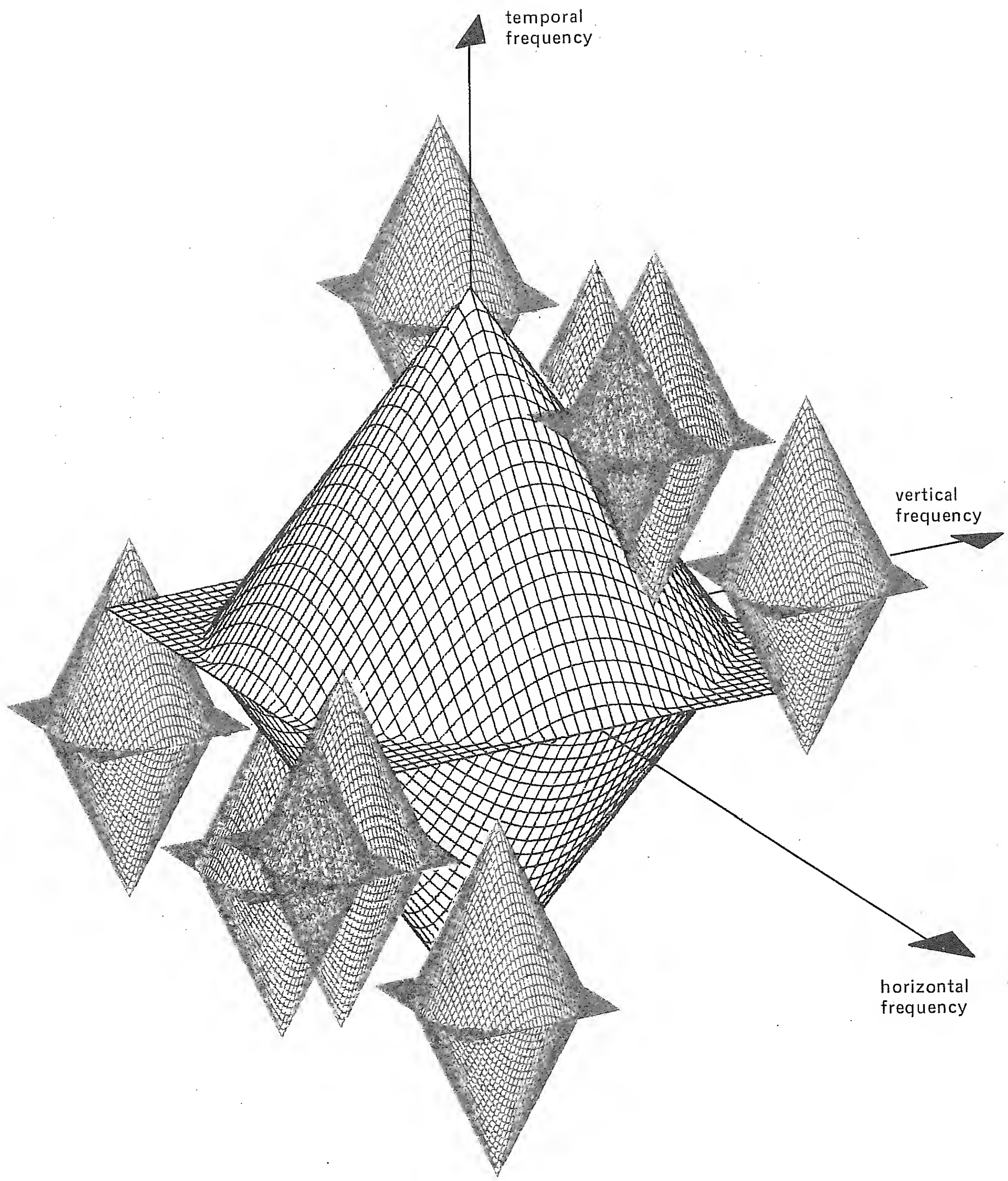


Fig. 29 - Perspective view of the ideal PAL signal spectrum

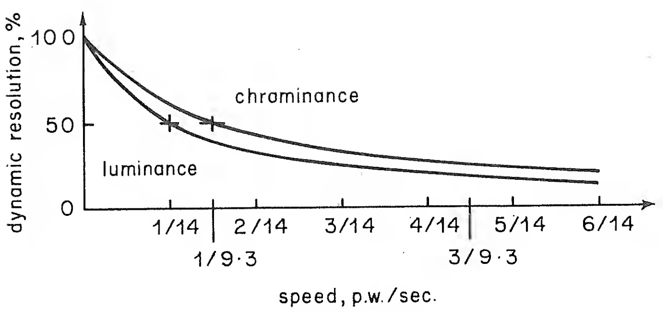


Fig. 30 - Relative dynamic resolutions of the luminance and chrominance in the ideal model, as percentages of the static resolution

this corresponds to 78 c./p.h. which is one-quarter of the theoretical maximum vertical-resolution. The maximum temporal resolution of the chrominance cannot be 25 Hz because it would then interfere with the luminance. Suppose, therefore, that it is limited to 12·5 Hz; this would mean that the chrominance dynamic resolution would still decrease more slowly than the luminance resolution. Both dynamic resolutions are shown in Fig. 30.

If the luminance and chrominance surfaces have these dimensions a section of the three-dimensional frequency space perpendicular to the m axis and centred on the value 284 (roughly 4·43 MHz) would appear as in Fig. 31. With the values chosen, the luminance and chrominance areas do

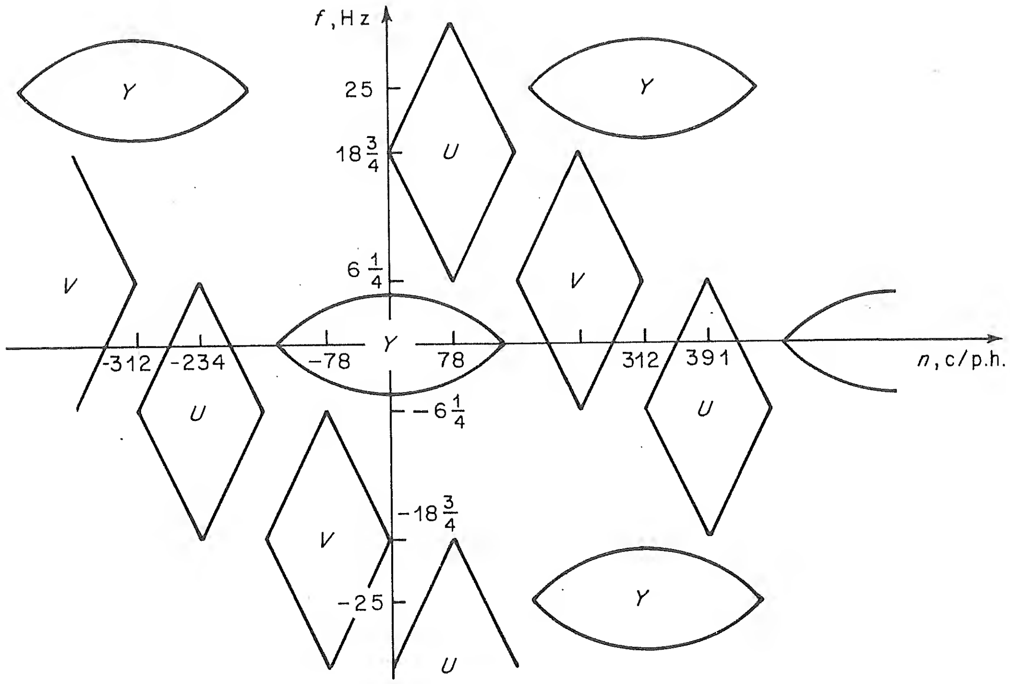


Fig. 31 - Section of the PAL signal spectrum at a value of m corresponding to 4·43 MHz

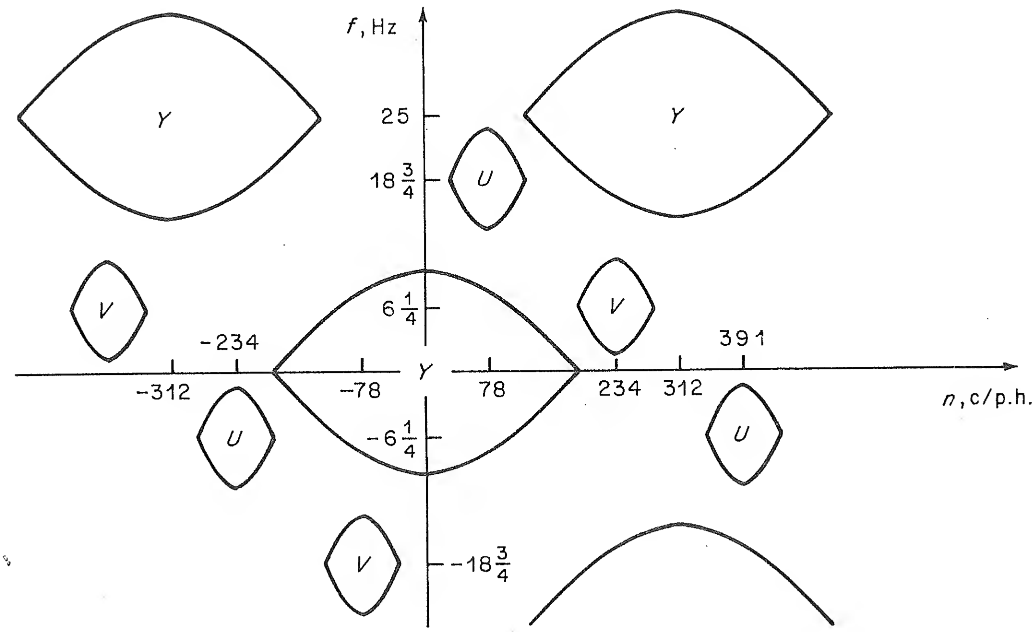


Fig. 32 - Section of the PAL signal spectrum at a value of m corresponding to 3·3 MHz

not overlap here. Moreover, they do not interact for other values of m. As m decreases, the luminance area grows, showing that faster movement is allowed, but the chrominance areas shrink from their maximum values of Fig. 31. Fig. 32 shows a cross-section centred on an m value of 213 (roughly 3·3 MHz). As can be seen, the separation of the luminance and chrominance areas is about the same as in Fig. 31. These figures illustrate that, in terms of the above assumptions above the visual process, there is ample room, in three-dimensional terms, for the colour spectral energy.

5.7. Possible spatio-temporal filters

Only the simplest ones will be discussed here, but it is easy to see how the method of derivation can be extended. Fig. 31 defines more precisely the areas of maximum and minimum response at an m value of 284. The variation with m can be achieved by cascading the vertical-temporal filter with a horizontal i.e. simple bandpass, filter. Although this is a variables-separable approach for m, experience with low-order filters discussed previously suggests that they

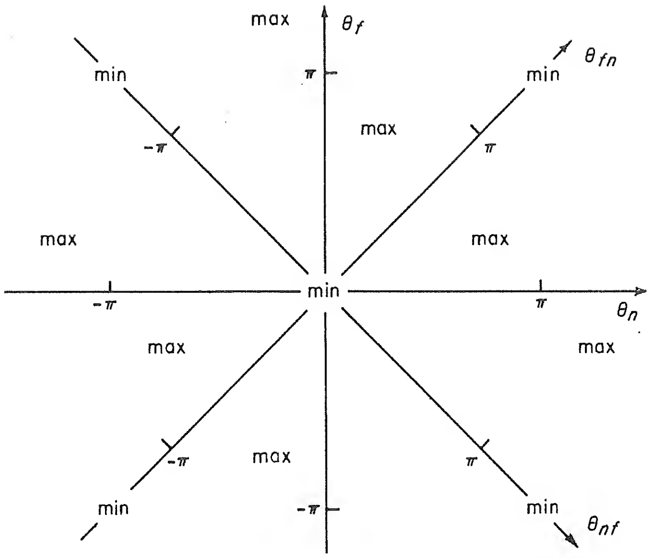


Fig. 33 - Section of the three-dimensional characteristic of Fig. 27 expressed in terms of normalised variables

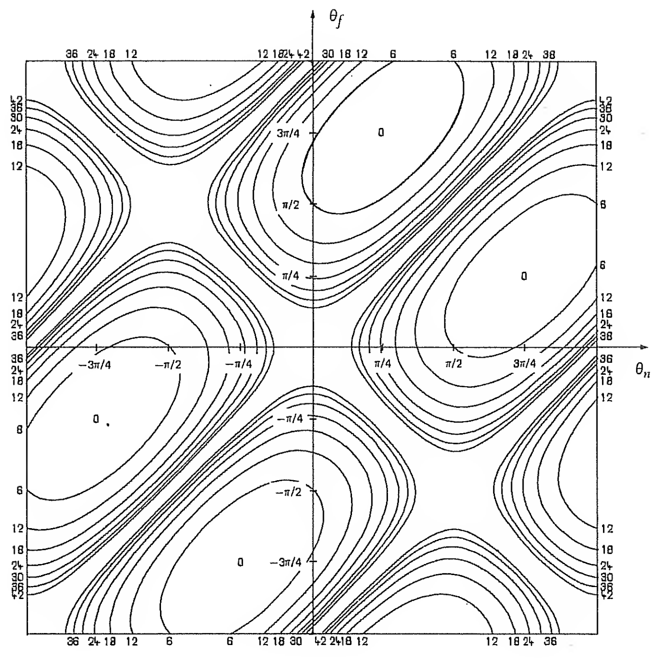


Fig. 34 - Section of the characteristic of the simplest filter that separates U, V and Y

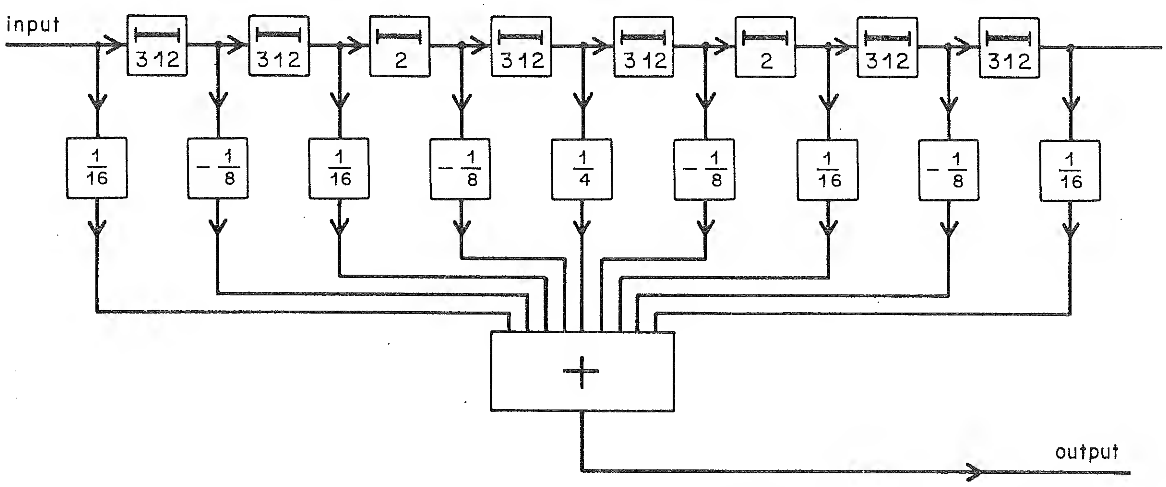


Fig. 35 - The realisation of the filter having the spectral characteristic of Fig. 34. The numbers are amounts of delay in line periods

behave fairly isotropically. But clearly the required *n-f* characteristic is not variables-separable and cannot be obtained by cascading vertical and temporal filters.

The required characteristic can be re-defined in terms of normalised variables θ_n and θ_f as shown in Fig. 33. These are defined in terms of the vertical-temporal sampling lattice of the interlace system, shown in Fig. 22. In this treatment the temporal skew of the vertical points is ignored. The definitions of θ_n and θ_f then become

$$\theta_n = 2\pi n Y$$

$$\theta_f = 2\pi f T$$

where Y is the vertical pitch of the picture lines measured in picture heights and T is the field period (for System I these quantities are 625^{-1} and 50^{-1}).

At first sight the realisation of a function like that of Fig. 33 appears difficult. The problem can be greatly simplified however, by rotating the axes through 45° as shown. In terms of the new variables θ_{nf} and θ_{fn} the ideal function is now considerably easier to express and amounts to the product of square-wave variations in both axes; the period in the θ_{nf} direction is, however, half that in the θ_{fn} direction. This means that the Fourier coefficients of the filter are spaced twice as far apart in the *y-t* direction compared with the *t-y* direction.

The simplest function that gives a pattern of maxima and minima as shown in Fig. 33 is the product of cosine variations along both axes, the frequency along *n-f* being twice that along *f-n*. Mathematically this is given by

$$\tfrac{1}{4}(1 - \cos 2\sqrt{2}\,\theta_{nf})\,(1 - \cos \sqrt{2}\,\theta_{fn})$$

TABLE 3

Spatio-Temporal Filter Coefficient Arrays

Separate *U-V* filter

	LINES						
FIELDS	0		1/16		0		0
		−1/8		0		0	
	1/16		0		−1/8		0
		0		1/4		0	
	0		−1/8		0		1/16
		0		0		−1/8	
	0		0		1/16		0

Combined *U-V* filter

	LINES		
FIELDS	0		−1/4
		1/2	
	−1/4		0

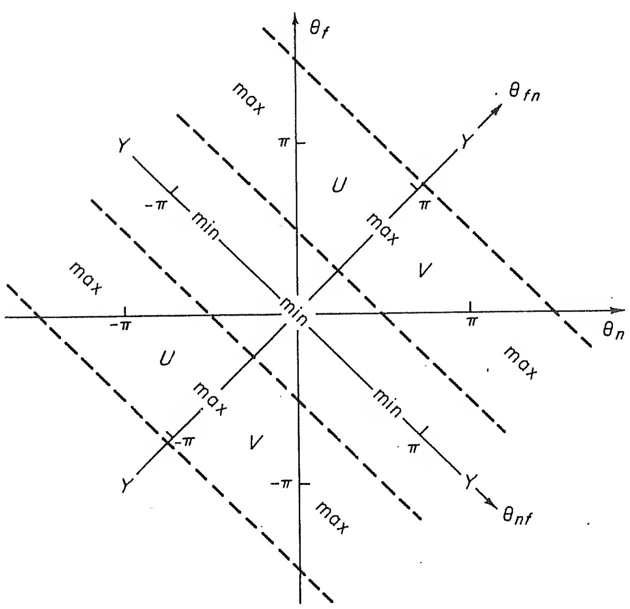


Fig. 36 - Outline of filter characteristic which separates U and V from Y

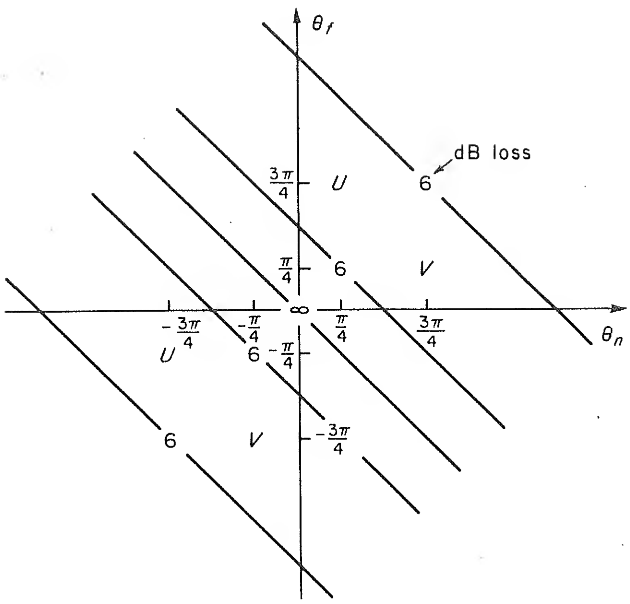


Fig. 37 - Section of the characteristic of the simplest filter which separates U and V from Y

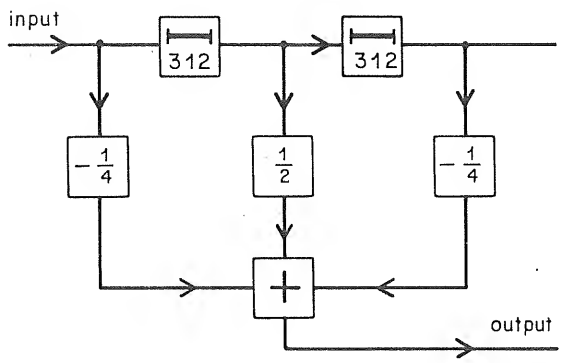
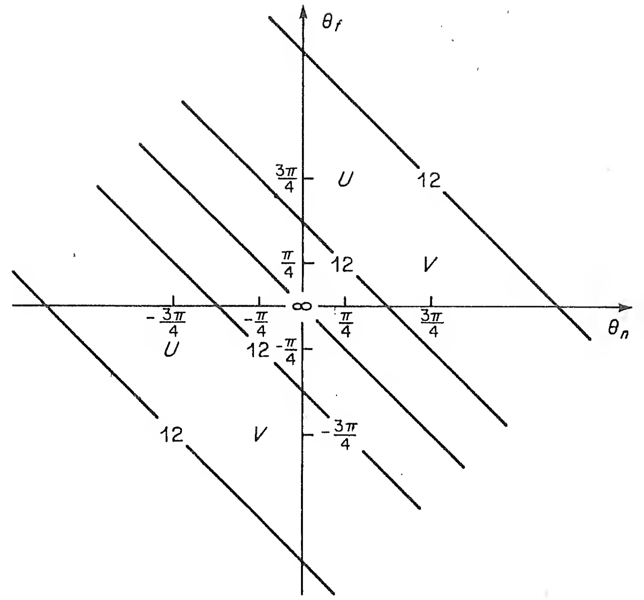


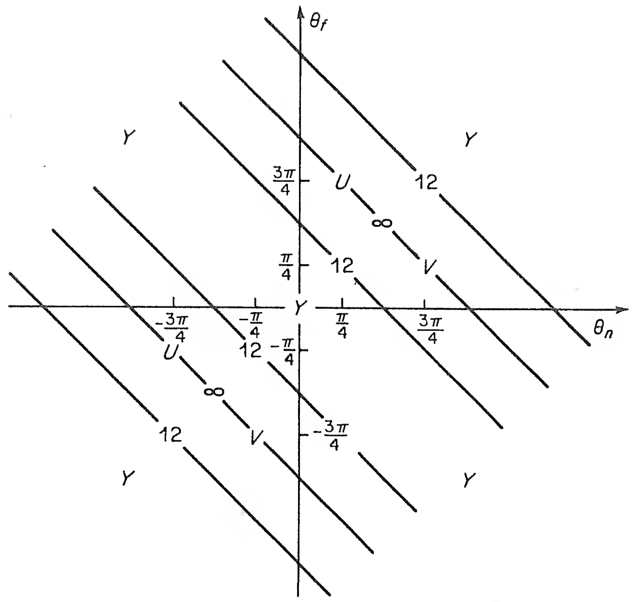
Fig. 38 - The realisation of the filter having the spectral characteristic of Fig. 37

The function is shown in Fig. 34. Comparing Fig. 31 and Fig. 34 it can be seen the areas of maximum response corresponding to the chrominance areas, are elongated in the θ_{fn} direction and compressed in the θ_{nf} direction; this is a natural consequence of trying to filter out the U and V areas separately. The pattern of weighting coefficients corresponding to this filter is shown in Table 3. The greater resolution in the θ_{nf} direction is reflected in the greater coefficient spacing in the y-t direction. The pattern extends in time over six field periods and vertically over six picture lines. Such a filter would be realised by the arrangement of Fig. 35. A compensating delay of 938 lines would also be necessary.

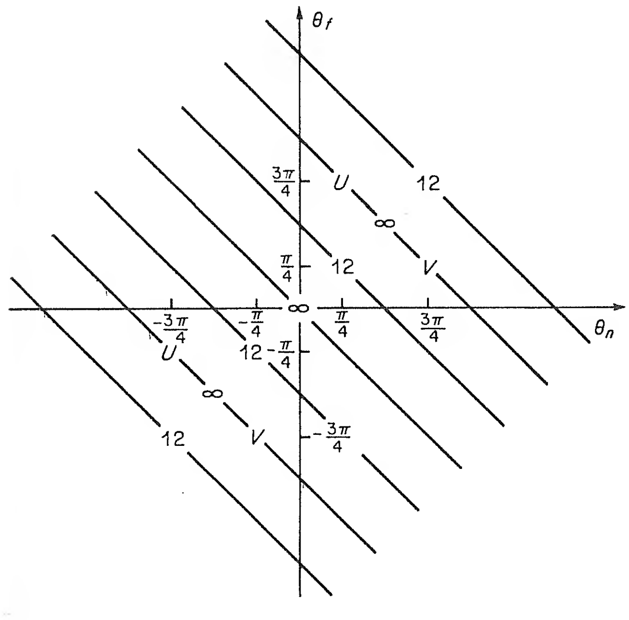
An even simpler solution is to avoid separating the U and V areas at the expense of losing the luminance in the region between them. According to the visual model, the loss of this luminance information is of little consequence. The required filter response is then a continuous maximum band in the θ_{nf} direction as shown in Fig. 36. The simplest function which satisfies this condition is given by the second term of the previous expression and is shown in



(a)



(b)



(c)

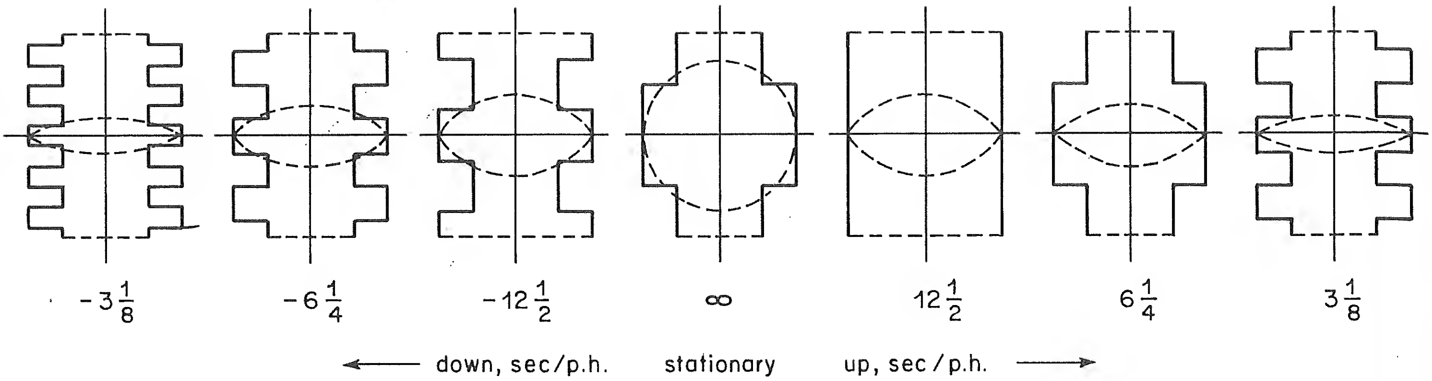
Fig. 39 - Section of the matched system spectral characteristics using the filter of Fig. 38
(*a*) chrominance (*b*) luminance (*c*) cross-signals

Fig. 37; the coefficient pattern for the filter is included in Table 3. As can be seen, the pattern now extends over only two field periods. The realisation of such a filter is shown in Fig. 38; a compensating delay of 312 lines would be necessary.

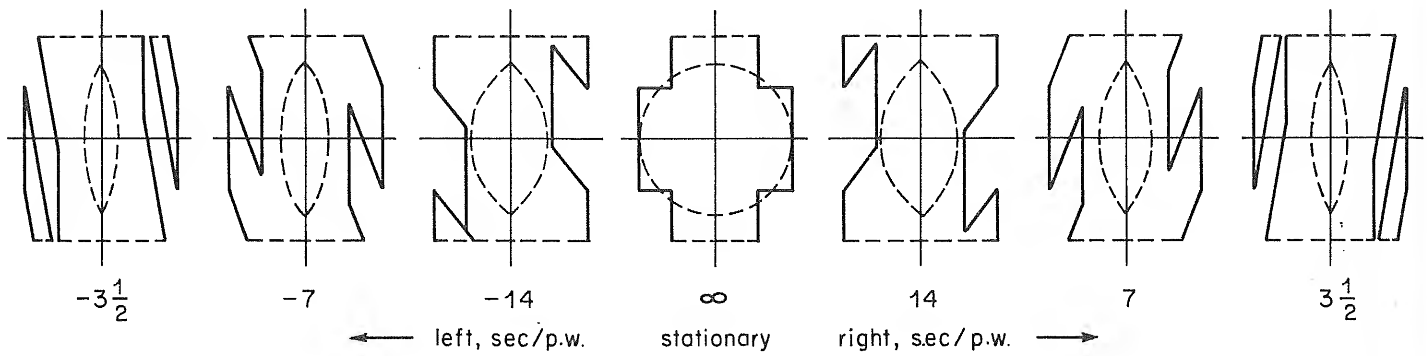
The matched-system chrominance, luminance and cross-signal characteristics are shown in Fig. 39. Compared with the intra-field filters it can be seen that these characteristics represent a considerable improvement. For example, the stationary chrominance vertical-bandwidth down to the 12 dB points is now plus or minus one-half the theoretical maximum compared with plus or minus one eighth for the intra-field filters.

Compared with the idealised model of the PAL signal, exemplified in Fig. 31, the characteristic of this filter is generous. For example, the channel of unity response in the θ_{nf} direction means that pure upward vertical motion of 12½ sec./p.h. is not spatially filtered at all.

The dynamic resolution of the luminance down to the −12 dB level for various motions is shown in Fig. 40. This should be compared with Fig. 24 and is plotted over the same range. The dotted areas show the dynamic resolution corresponding to the ideal model. Assuming an ideal band-pass filter of, say, ±1·3 MHz, the resolution is, of course, unaffected below 3·1 MHz. Above 3·1 MHz certain spatial frequencies are lost or attenuated, the pattern of loss



(a)



(b)

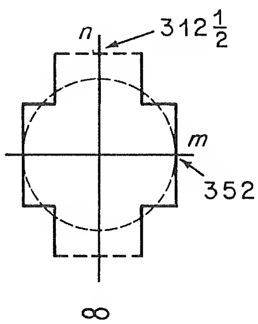


Fig. 40 - Dynamic luminance resolution of the matched system down to the −12 dB level using the filter of Fig. 38 (in conjunction with an ideal bandpass filter)

(a) vertical motion (b) horizontal motion

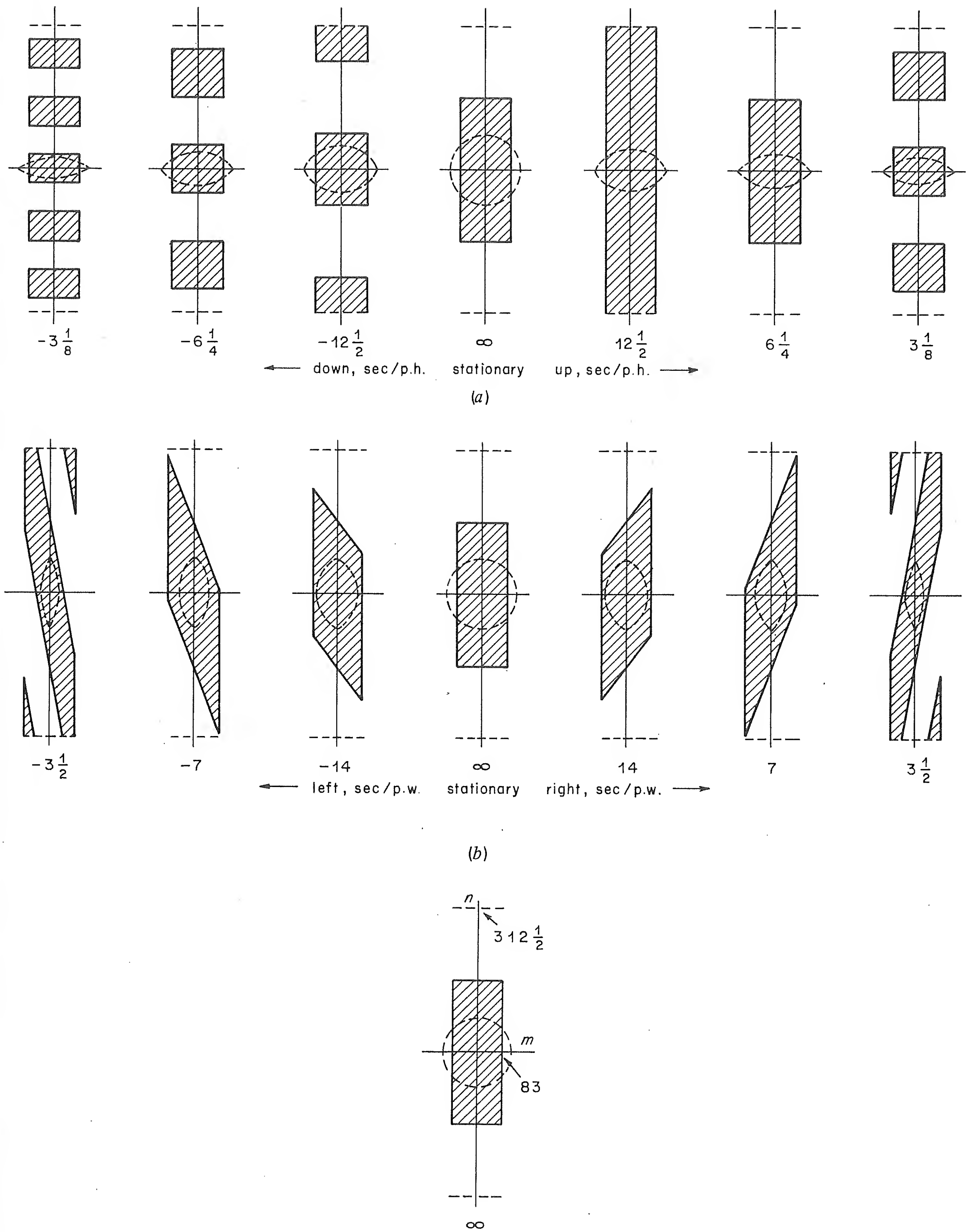


Fig. 41 - Dynamic chrominance resolution of the matched system under the same conditions as in Fig. 40
(*a*) vertical motion (*b*) horizontal motion

depending on the amount and direction of movement. Stationary pictures have a resolution which is nearer the isotropic ideal than that given by the intra-field filters. Pure upward motion increases the vertical resolution at first, pure downward motion decreases it. Pure horizontal motion gives a skew sinusoidal pattern of attenuation. For fast horizontal motion the striations of the pattern become nearly perpendicular to the m axis.

The dynamic resolution of the demodulated chrominance is shown in Fig. 41 over the vertical range ±312½ c./p.h. The behaviour is generally the same as for the luminance except that it is centred on the origin. Stationary pictures have a rectangular resolution characteristic with a vertical limit of ±156¼ c./p.h. at the −12 dB points. Pure upward motion increases the limit at first, pure downward motion decreases it, and pure horizontal motion gives a skew pattern as before. Severe motion causes the spectrum to break up into bands.

The subjective effect of this dynamic resolution is uncertain. However, it can be confidently stated that with the visual model of Section 5.6 as a guide, the subjective impairment will be minimal. The curious effects will be outweighed by the eye's loss of dynamic resolution.

5.8. Filtering at only one end of the system

If such a filter is used only at the coder, then the decoder chrominance circuits will accept signals from all the spectral space and cross-colour will be produced by the residual luminance. The luminance removed at the coder will be that which causes the slow moving cross-colour on stationary pictures, since it lies near the chrominance carriers; however, the fast-moving cross-colour will remain. With moving pictures, in which the movement is such that it would have caused slow-moving cross-colour, then the cross-colour is removed.

Equally, the decoder luminance circuits will accept all signals so that cross-luminance will be produced by the residual chrominance. The chrominance removed at the coder will be that which causes the coarse vertical-frequency, slow-moving cross-luminance. But the fast-moving and fine vertical-frequency components remain. In particular the chrominance carriers themselves are present so that, as with the intra-field filters, a conventional notch filter must be included in the decoder luminance path.

If the filter is used only at the decoder then it is the fast-moving components that are eliminated. Thus the slow-moving cross-colour and slow-moving coarse cross-luminance remain. In particular, the chrominance carriers are rejected so that a luminance notch filter is unnecessary.

The effects of the permutations of filter position are shown in Table 4.

5.9. Summary

With the aid of a three-dimensional spectral model a form of a filter has been derived which, when used in a matched system, conveys the maximum amount of information in the luminance and chrominance channels without interaction. For stationary pictures the luminance resolution is somewhat better than with the intra-field filters and the chrominance resolution is considerably better. For moving pictures the loss of resolution is thought to be significantly less than that occurring in the eye.

TABLE 4

		CODER	
		No filtering	Filtering
DECODER	No filtering (other than notch filter)	1	2
DECODER	Filtering	3	4

1. Present situation.

2. Satisfactory luminance diagonal and chrominance vertical resolution on stationary pictures. Slow moving cross-colour eliminated. Fast-moving cross-colour and cross-luminance of high amplitude remains on stationary pictures. (Thus subcarrier luminance patterning is present in full amplitude unless an additional luminance notch is present in the decoder.) Certain forms of movement have reduced spatial resolution.

3. Satisfactory luminance diagonal and chrominance vertical resolution, on stationary pictures. Fast-moving cross-colour eliminated. Slow-moving cross-colour and cross-luminance of low amplitude remain on stationary pictures. Certain forms of movement have reduced spatial resolution.

4. Satisfactory luminance diagonal and chrominance vertical resolution on stationary pictures. No cross-colour. Certain forms of movement have reduced spatial resolution.

6. A simplification for the matched system

A certain simplification is possible, within the confines of a matched system like that of Fig. 3, if the filters used are first-order, that is, having a $\cos^2$ frequency characteristic. This is simply to assume that the $\cos^2$ function describes the overall characteristic rather than that of the individual filters. This means that the individual filters have a cosine characteristic which can be realised by taking equal weights across a single delay element. Thus a system based on two-dimensional filters becomes as shown in Fig. 42(a), whilst one based on three-dimensional filters becomes as shown in Fig. 42(b); thus not only is the filter delay halved but also the large compensating delays are unnecessary.

The difficulty with this type of filter is that it is not phase-corrected; this is because the group delay of the filter corresponds to a point that lies vertically midway between the scanning lines. However, when two such

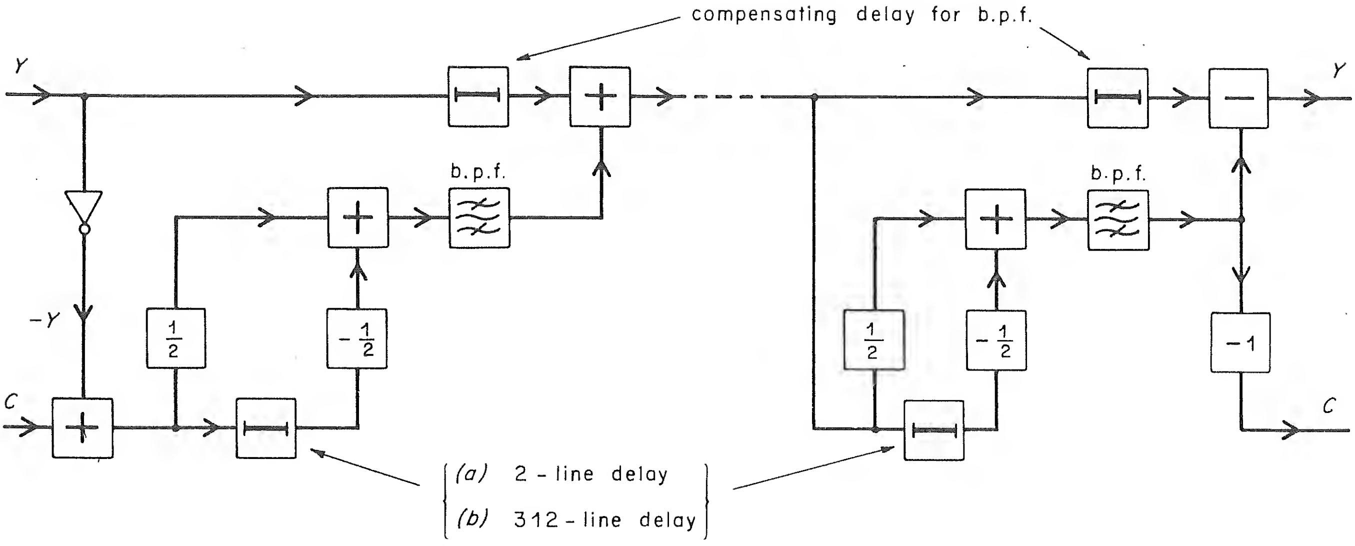


Fig. 42 - Simplified form of matched system using non-phase-corrected filters
(*a*) two-dimensional (*b*) three-dimensional

filters are cascaded the resultant is phase-corrected. Thus the filter can be used provided that it is acceptable to have a non-phase-corrected signal at the coder-decoder interface. Further, the cross-signal level has a maximum amplitude of −6 dB instead of −12 dB. The potential cost-saving using a filter such as that shown in Fig. 42(*b*) would, of course, be considerable, when compared with those outlined in Figs. 35 and 38.

7. Conclusions

The present methods of coding and decoding the PAL signal are not as effective as they could be in combining and separating the luminance and chrominance signals. As a result, mutual interference occurs which can be very objectionable for certain scenes. This Report has studied methods whereby luminance and chrominance are confined to distinctly separate frequency bands before being combined, and are separated by suitable filters so that interference cannot occur. The filters which characterise these methods can be based on line stores (intra-field processing) or field stores (inter-field processing).

Using intra-field filters it is possible to obtain an increased chrominance bandwidth as compared to that obtained at present and still preserve high-frequency luminance information; the conventional decoder notch filter is replaced by a more selective bandstop filter. The price paid is an additional loss of vertical chrominance resolution, but it is argued that such resolution is excessive at present and the loss would be noticed only on test signals containing synthetic vertical detail.

Using inter-field filters it is possible to obtain a further increase in chrominance bandwidth and to preserve yet more of the luminance information at the expense of limitations on the spatial spectra of moving objects which are determined by the velocities with which the objects move. From published information on the characteristics of human vision, these limitations do not appear to be serious; nevertheless such information should be interpreted with some caution.

Although the potential advantages of the proposed methods rely upon the use of special filters at both the coder and decoder, some benefit would nevertheless be obtained by the use of a filter at only the coder. It is, however, possible that a relatively cheap solution based on simplified intra-field filters, could be used at both the coder and decoder. In this case the characteristics of the coder filter would be such that compatibility with existing decoders would need to be investigated.

The potential advantages of the proposed methods are immediately relevant to the transcoding of PAL to other colour standards; this, of course, would involve the incorporation of special filters in all PAL coders. Further, the introduction of more complex techniques, such as sub-Nyquist sampling, for digital coding, might benefit greatly from the more rigid spectral specification which the inter-field method implies.

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